MESONIC CHIRAL LAGRANGIAN MODELS WITH ISOSPIN BREAKING

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By
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CERTIFICATE

Certified that the work presented in this thesis entitled, 'Mesonic Chiral Lagrangians with Isospin Breaking', by A.K. Kapoor has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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SYNOPSIS

Thesis entitled, 'Mesonic Chiral Lagrangian Models with Isospin Breaking', submitted by ASHOK KUMAR KAPOOR in partial fulfilment of the requirements of the Ph.D. degree to the Department of Physics, Indian Institute of Technology, Kanpur

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In this work we report on some investigations of two Lagrangian models of broken chiral symmetry. The models provide a unified framework for calculating various dynamical quantities including particle masses. The first of the two models is discussed in detail and forms the main part of the thesis. In this model the chiral $U(3) \otimes U(3)$ symmetry is chosen to be the basic symmetry for the Lagrangian. Isospin violating effects are also included. The Lagrangian and the vacuum are thus invariant under $U(1)_{I_3} \otimes U(1)_Y \otimes U(1)_{V_0}$ only. In the second model all symmetry breaking effects are obtained from spontaneous breaking alone.

The thesis is divided into six chapters. The first chapter is an introduction. In the second chapter the basic fields are introduced and assigned to suitable representations. These are nonets of spin $0^{\pm}$ fields and vector and axial vector gauge fields which are assumed to transform as $(3,3^*) \oplus (3^*,3)$ and $(1,8) \oplus (8,1)$ representations respectively. The Lagrangian is chosen to be invariant under $U(3) \otimes U(3)$ gauge transformations

except for two terms. The first one, a common mass term for gauge fields, is invariant under group transformations with constant parameters. The second term breaks the symmetry explicitly and is assumed to transform as a component of the $(3,3^*) \oplus (3^*,3)$ representation. Polar decomposition of the fields is performed and the physical fields are defined after considering field mixings and renormalizations. The particle masses are obtained in the tree approximation and most of the parameters of the theory are fixed through some of the experimental masses.

The effective Lagrangian obtained in the second chapter is used to study mesonic interactions in the third chapter. The rates for two body decays $V \to PP$, $A \to VP$, $S \to PP$ and $A \to SP$ and for three particle decays $A \to 3P$ and $X^0 \to \eta\pi\pi$ are studied. The scattering lengths and effective ranges for meson-meson scattering processes have been studied.

In the fourth chapter the structure of currents is studied. The mesonic decay constants F_K, F_π, F_{K^*}, f_ρ, f_ω, f_ϕ and the K_{l3} decay form factors are discussed.

The isospin violating effects on mass differences and the decays $\rho \to \eta\pi$, $\omega \to \pi^+\pi^-$ and $\eta \to 3\pi$ are studied in the fifth chapter.

In the sixth and last chapter we study a renormalizable model of broken chiral symmetry. The most important feature of this second model is that it is fully gauge invariant and renormalizable. The symmetry breaking effects including the masses of all gauge fields are induced by vacuum breaking. This is achieved by introducing three sets of auxiliary scalar fields each transforming as $(3,1) \oplus (1,3)$ representation of $SU(3) \otimes SU(3)$. In addition to the usual vector and axial vector gauge fields the photon is introduced in a natural way and the induced $\rho - \omega$ and $\rho - \phi$ mixings are studied. The rate for $\omega \to \pi^+\pi^-$ is found to be in agreement with the experiments in contrast with the first model where $\Gamma(\omega \to 2\pi)$ was rather large. The results on the lepton pair decays of the neutral vector mesons are also in better agreement in this model.

CHAPTER I

INTRODUCTION

Current Algebra[1] with vector meson dominance[2] and PCAC assumptions[3] has been successfully applied to a number of processes. It was noted by Weinberg[4] that current algebra results could be obtained from the σ-model[5] by taking the limit $m_\sigma \to \infty$. It was subsequently observed by many authors[6] that suitably constructed nonlinear Lagrangians also give current algebra results in the tree graph approximation. The Lagrangian approach has the advantage of being simple and self consistent. Many assumptions made in current algebra calculations can be tested in specific models. In the conventional Lagrangian approach[7] one is forced to introduce particles whose existence is not confirmed or whose properties are not known in detail. For example , the SU(3) version of σ-model[8] involves a nonet of scalar particles. When there was little evidence for the existence of the scalar mesons, chiral invariant Lagrangians with only pseudoscalar mesons were constructed. This was achieved by assuming that the pseudoscalar fields transform linearly under SU(3) transformations but nonlinearly under chiral transformations[9]. It was noticed by many authors[10] that effective nonlinear Lagrangians could be generated from the conventional linear Lagrangians by making chiral partners of the 'would be' Goldstone bosons infinitely heavy. As the evidence

for the existence of scalar mesons is increasing there is no need to eliminate them from the theory.

It is very clear th.t chiral SU(3) X SU(3) is not an exact symmetry, of hadrons since, for example, the axial vector currrents are known to be non-conserved. Either an explicit symmetry breaking term or vacuum breaking or both may be used to obtain necessary symmetry breaking effects. If symmetry is broken only by the vacuum the Goldstone theorem[11] implies that there must exist some massless particles - the Goldstone bosons. The Goldstone theorem, however, fails to apply if the symmetry is extended to a gauge symmetry and the Lagrangian is made fully gauge invariant by introducing an appropriate number of gauge fields[12]. In this case spontaneous breaking manifests itself through some of the gauge fields acquiring mass. The Goldstone bosons become longitudinal·components of the gauge fields and disappear from the theory. Until very recently this mechanism - the Higgs Kibble mechanism - could not be used successfully to generate physical masses of the vector and axial vector fields in the case of SU(3) X SU(3) model of strong interactions[13]. In particular gauge fields coupled to conserved currents remained massless. To cure this a common mass term for gauge fields was added which, being gauge non-invariant, prevented the elimination of massless Goldstone bosons which inturn, were made massive by adding an explicit symmetry breaking term in the Lagrangian[14].

The explicit symmetry breaking term was usually chosen to transform as a component of the $(3,3^*) + (3^*,3)$ representation[15]. Other simple choices of symmetry breaking term transforming as a component of $(8, 8)$[16] $(1,8) + (8,1)$[17] or $(6, 6^*) + (6^*, 6)$[18] have also been studied.

The first of the two models[19] studied in the present work is a phenomenological Lagrangian model of broken chiral U(3) X U(3) symmetry and forms the main part of the thesis. The masses and interactions of spin zero and spin one mesons are studied in tree approximation. Both the Lagrangian and the vacuum are assumed to be noninvariant under the group transformations. The symmetry breaking is, at first, assumed to respect the isospin symmetry but later the isospin breaking effects are also studied. Since there is growing evidence[20] for the existence of scalar mesons, we donot follow the approach of Coleman Wess and Zumino[21] of constructing the chiral invariant nonlinear Lagrangians directly from pseudoscalar fields alone; instead we start with a Lagrangian written in terms of linear ly transforming scalar and pseudoscalar fields. The Lagrangian is assumed to have the following features: The Lagrangian is fully gauge invariant except for a common mass term for the gauge fields and an explicit symmetry breaking term. The common mass term is invariant under group transformations with constant parameters only. The explicit symmetry breaking term is assumed to be linear in the scalar fields and transforms as a component of the $(3, 3^*) + (3^*, 3)$.

representation. We would like to have the gauge fields coupled minimally to other fields. However, in order to explain the vector and axial vector masses and their decay widths in the tree approximation some non-minimal interaction terms are included in the Lagrangian. An effective nonlinear Lagrangian is generated by performing polar decomposition[22] of the fields and is then used to study various processes.

While the calculations in the first model were nearing completion it was learnt that it is possible to obtain a common mass term for gauge fields through Higgs-Kibble mechanism[23]. In the last chapter a model of chiral symmetry is discussed in which all realistic symmetry breaking effects, including physical masses for gauge fields, are obtained through spontaneous breaking alone[24]. The photon is included in a natural way in this second model. The inclusion of photon induces $\rho - \omega$ and $\rho - \phi$ mixings which are estimated after fixing most of the parameters of the theory from particle masses. An important feature of this model is that the Lagrangian is renormalizable[25] and only $(3, 3^*) + (3^*, 3)$ and $(1, 8) + (8, 1)$ type of symmetry breaking effects are present in the hadronic interactions. Since couplings are restricted to renormalizable type, the nonminimal couplings are not included. The decays of the spin one fields cannot be described correctly in the lowest order. The higher order graphs may produce necessary corrections. However, we donot calculate the decays and higher order corrections to them.

The plan of the thesis is as follows. In the second chapter the basic fields are introduced and are assigned suitable representations. The basic Lagrangian for the first model for the nonets of spin zero and spin one mesons is written. Polar decomposition of the fields is performed and the physical fields are defined after considering field mixings and renormalizations. Most of the parameters are fixed through some of the particle masses and the remaining masses are predicted. The effective Lagrangian obtained in the second chapter is used to study mesonic interactions in the third chapter. The rates for the two body decays $V \to PP$, $A \to VP$, $S \to PP$ and $A \to SP$ and for three particle modes $A \to 3P$ and $X^0 \to \eta\pi\pi$ are calculated in the tree approximation. Finally the scattering lengths and effective ranges for $\pi - \pi$, $K - \pi$ and $K - K$ scattering processes are calculated. The fourth chapter is devoted to study of structure of currents, the mesonic decay constants and the K_{13} form factors. In the isospin violating effects on mass differences and the decays $\rho \to \eta\pi$, $\omega \to \pi^+\pi^-$ and $\eta \to 3\pi$ are studied in the fifth chapter. The sixth and last chapter the Lagrangian for the second model is written down and particle masses are calculated and the $\rho - \omega$ and $\rho - \emptyset$ mixings are studied.

CHAPTER II

THE EFFECTIVE LAGRANGIAN AND PARTICLE MASSES

In this chapter the basic Lagrangian for spin zero and spin one meson fields is written. Some of the scalar fields are assumed to have non zero vacuum expectation values. An effective nonlinear Lagrangian is generated by performing the polar decomposition of the fields. Field mixings and renormalizations are considered and the physical fields are defined. The particle masses are obtained in tree approximation. Some of the experimental masses are used to fix all except two parameters of the model. The scalar meson masses are predicted.

2.1 Fields and the Lagrangian:

The basic symmetry group of the model is chiral U(3) X U(3) a general transformation of which is represented by the unitary operator,

$$\mathcal{U} = \exp\left(- i \sum_{k=0}^{8} \left(\epsilon_k^L Q_k^L + \epsilon_k^R Q_k^R \right)\right).$$

The meson matrix,

$$M' = \frac{1}{\sqrt{2}} \sum_{k=0}^{8} \lambda_k \left(u_k + i v_k\right),$$

of scalar and pseudoscalar meson fields u_k and v_k belonging to the representation $(3,3^*) + (3^*,3)$, transforms as

$$\mathcal{U}\, M\, \mathcal{U}^+ = U_L\, M\, U_R^+$$

where,

$$U_L = \exp\left(i \sum_{k=1}^{8} \epsilon_k^L \lambda_k/2\right) \exp\left(i\epsilon_o^L \lambda_o/2\right)$$

$$\equiv \hat{U}_L \; U_L^o$$

and

$$U_R = \exp\left(i \sum_{k=1}^{8} \epsilon_k^R \lambda_k/2\right) \exp\left(i\, \epsilon_o^R \lambda_o/2\right)$$

$$\equiv \hat{U}_R \; U_R^o \, .$$

The symmetry is extended to a gauge symmetry by introducing the gauge fields $X_\mu^{L,k}$, $X_\mu^{R,k}$, $(k = 0,1,\ldots, 8)$ transforming as,

$$\mathcal{U}\, X_\mu^L\, \mathcal{U}^+ = \hat{U}_L\, X^L\, \hat{U}_L^+ - \frac{1}{ig}\, \hat{U}_L\, \partial_\mu\, \hat{U}_L^+$$

$$\mathcal{U}\, X_\mu^{L,o}\, \mathcal{U}^+ = X_\mu^{L,o} + \frac{1}{g_o}\, \partial_\mu\, \epsilon_o^L$$

where,

$$\hat{X}_\mu^L = \frac{1}{2} \sum_{k=1}^{8} X_\mu^{L,k}\, \lambda_k$$

with similar equations for $\hat{X}_\mu^R$ and $X_\mu^{R,o}$. The parameters g and g_o are coupling constants for the octet and singlet gauge fields. In order to preserve parity the coupling constants for the left handed and the right handed gauge fields are chosen to be same.

The Lagrangian density is taken to be the following:

$$L = L_M + L_{V,A} + L_{non.min.} + L_{S.B.} \tag{2.1}$$

$$L_M = -\frac{1}{2}\, \mathrm{Tr}\,(D_\mu M^+ D_\mu M) - \frac{1}{2}\, \mu_o^2\, (W_2 + \nu_1 W_4 + \nu_2 W_2^2\,) \tag{2.2}$$

where,

$$D_\mu M = \partial_\mu M - ig\,(\hat{X}^L_\mu\; M - M\; \hat{X}^R_\mu\,) -$$

$$- ig_o\, \frac{\lambda_o}{2}\,(X^{L,o}_\mu - X^{R,o}_\mu)\, M$$

$$W_2 = \mathrm{Tr}\,(M^+ M)\,, \qquad W_4 = \mathrm{Tr}\,(M^+ M\, M^+ M)$$

$$\begin{aligned} L_{V,A} = & -\frac{1}{4}\,\mathrm{Tr}\,(X^L_{\mu\nu}\; X^L_{\mu\nu} + X^R_{\mu\nu}\; X^R_{\mu\nu}\,) \\ & -\frac{1}{2}\, m_o^2\, \mathrm{Tr}\,(X^L_\mu\; X^L_\mu + X^R_\mu\, X^R_\mu\,) \\ & -\frac{1}{12}\; m_A^2\,(\mathrm{Tr}\,(X^L_\mu - X^R_\mu\,))^2 \end{aligned} \qquad (2.3)$$

where,

$$X^L_\mu = \frac{1}{2}\sum_{k=0}^{8} X^{L,k}_\mu\; \lambda_k = \hat{X}^L_\mu + \frac{1}{2}\;\lambda_o\; X^{L,o}_\mu$$

$$X^L_{\mu\nu} = \partial_\mu X^L_\nu - \partial_\nu\, X^L_\mu - ig\,[X^L_\mu\,, X^L_\nu]$$

and we have similar expressions for X^R_μ and $X^R_{\mu\nu}$.

$$\begin{aligned} L_{non.min.} = &\; i\, h_o\;\mathrm{Tr}\,(X^L_{\mu\nu}\;\; D_\mu M\, D_\nu M^+ + X^R_{\mu\nu}\;\; D_\mu M^+\, D_\nu M) \\ & -\frac{h_1}{4}\,\mathrm{Tr}\,(X^L_{\mu\nu}\;\; X^L_{\mu\nu}\;\; MM^+ + X^R_{\mu\nu}\;\; X^R_{\mu\nu}\; M^+ M) \\ & -\frac{h_2}{2}\,\mathrm{Tr}\,(X^L_{\mu\nu}\; M\; X^R_{\mu\nu}\; M^+) \end{aligned} \qquad (2,4)$$

The first term in $L_{non.min.}$ is needed to explain the decays of **axial** vector mesons while the last two terms are necessary to obtain physical masses for the vector and axial vector mesons.

$$L_{S.B.} = \frac{\lambda}{2}\,(\det M + \det M^+) + \frac{1}{2}\,\mathrm{Tr}\,(A\,(M + M^+)) \qquad (2.5)$$

where,

$$A = \frac{1}{\sqrt{2}} (a_o \lambda_o + a_8 \lambda_8 + a_3 \lambda_3)$$

$$\equiv \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \quad (2.6)$$

In $L_{S.B.}$ the first term is not invariant only under ninth axial transformations. The second term breaks the symmetry further to $SU(2)_I \times U(1)_Y \times U(1)_{V_o}$ and to $U(1)_{I_3} \times U(1)_Y \times U(1)_{V_o}$ in the two cases $a_3 = 0$ and $a_3 \neq 0$ respectively.

2.3 Field Mixings, Renormalizations and Particle Masses:

For the time being we ignore the isospin breaking effects and take $b = a$ ($a_3 = 0$) upto Chapter IV. In Chapter V we will consider the isospin breaking interactions. Assuming $\eta \equiv \langle M \rangle_o$ we can perform the following decomposition,[22]

$$M = e^{i\chi} e^{i\phi} (\eta + \Sigma) e^{i\phi} e^{-i\chi} \quad (2.7)$$

where,

$$\eta = \frac{1}{\sqrt{2}} (\eta_o \lambda_o + \eta_8 \lambda_8) \quad (2.8)$$

$$\equiv \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \gamma \end{bmatrix}$$

$$\Sigma = (\sigma_0 \lambda_0 + \sigma_1 \lambda_1 + \sigma_2 \lambda_2 + \sigma_3 \lambda_3 + \sigma_8 \lambda_8)/\sqrt{2} \quad (2.9)$$

$$\chi \equiv \begin{bmatrix} 0 & i\sigma_K/f\sigma_K \\ -i\sigma_K^+/f\sigma_K & 0 \end{bmatrix} \quad (2.10)$$

$$\emptyset = \frac{1}{\sqrt{2}} \sum_{k=0}^{8} \emptyset_k \lambda_k/f_{\emptyset_k} \quad (2.11)$$

where the new fields σ's and $\emptyset$'s are assumed to have zero vacuum expectation values.

Using the equations (2.7) – (2.11) the original fields u's and v's may be expressed as power series in the new fields χ's and $\emptyset$'s. For instance the pseudoscalar fields v's are given by,

$$v_k = \frac{1}{2i} \mathrm{Tr}((M - M^+) \frac{\lambda_k}{\sqrt{2}}) \quad (2.12)$$

Now,

$$M = e^{i\chi} e^{i\emptyset} (\eta + \Sigma) e^{i\emptyset} e^{-i\chi}$$

$$= e^{i\chi} \{ (\eta + \Sigma) + i [(\eta + \Sigma), \emptyset]_+ + \frac{i^2}{2!} [[\eta + \Sigma, \emptyset]_+, \emptyset]_+ + \dots \} e^{-i\chi}$$

and

$$M^+ = e^{i\chi} \{ (\eta + \Sigma) - i [\eta + \Sigma, \emptyset]_+ + \frac{i^2}{2!} [[\eta + \Sigma, \emptyset]_+, \emptyset]_+ + \dots \} e^{-i\chi}$$

$$\therefore \quad \frac{1}{2i}(M - M^{+}) = e^{ix}\{[\eta + \Sigma, \emptyset]_{+} - \frac{1}{3!}[[[\eta + \Sigma, \emptyset]_{+}, \emptyset]_{+}, \emptyset]_{+} + \ldots\} \; e^{-ix} \tag{2.13}$$

From (2.12) and (2.13),

$$v_k = \mathrm{Tr}\,(e^{ix}\frac{\lambda_k}{\sqrt{2}} e^{-ix}\{[\eta, \emptyset]_{+} + [\Sigma, \emptyset]_{+} - \frac{1}{3!}[[[\eta + \Sigma, \emptyset]_{+} i\emptyset]_{+}, \emptyset]_{+} + \ldots\}) \tag{2.14}$$

Assuming the transformation from u and v fields to σ and $\emptyset$ fields to be canonical the term linear in $\emptyset_k$ in Eq. (2.14) must have its coefficient equal to 1. This fixes up the constants $f_{\emptyset_k}$. The constant f_{σ_K} is similarly fixed by expanding u_K as a power series in σ and $\emptyset$ fields. Thus we have,

$$f_{\sigma_K} = 2(\alpha - \gamma) \tag{2.15}$$

and,

$$f_{\emptyset k} = \begin{cases} 2\alpha & \text{for} \quad k = 1,2,3 \\ \alpha + \gamma & \text{for} \quad k = 4, 5, 6, 7 \\ 2(\alpha + 2\gamma)/3 & \text{for} \quad k = 8 \\ 2(2\alpha + \gamma)/3 & \text{for} \quad k = 0 \end{cases} \tag{2.16}$$

Eq. (2.7) expresses M as a gauge transform of $(\eta + \Sigma)$. Performing the same transformation on the gauge fields and defining,

$$S_L = e^{ix} e^{i\emptyset}$$
$$S_R = e^{ix} e^{-i\emptyset} \qquad (2.17)$$

we have new gauge fields $Y_\mu^{L,k}$ and $Y_\mu^{R,k}$ given by,

$$\hat{X}_\mu^L = S_L \hat{Y}_\mu^L S_L^+ - \frac{1}{ig} S_L \partial_\mu S_L^+$$
$$X_\mu^{L,o} = Y_\mu^{L,o} - \frac{1}{2g_o} \sqrt{\frac{3}{2}} \; Tr\,(\partial_\mu \; \emptyset)$$
$$\hat{X}_\mu^R = S_R \hat{Y}_\mu^R S^+{}_R - \frac{1}{ig} S_R \partial_\mu S_R^+$$
$$X_\mu^{R,o} = Y_\mu^{R,o} + \frac{1}{2g_o} \sqrt{\frac{3}{2}} \; Tr\,(\partial_\mu \; \emptyset) \qquad (2.18)$$

Using equations (2.7) to (2.11) and (2.18) we obtain the Lagrangian for the new fields. In the part of the Lagrangian fully gauge invariant under coordinate dependent transformations, the fields M, X_μ^L and X_μ^R are replaced by $\rho \equiv (\eta + \Sigma)$, Y_μ^L and Y_μ^R, respectively. Thus we have, for the Lagrangian:

$$L = L_M + L_{V,A} + L_{non.min.} + L_{S.B.}$$

where L_M, $L_{V.A}$ etc., written in terms of new fields, are,

$$L_M = -\frac{1}{2} Tr\,(D_\mu \rho^+ D_\mu \rho)$$
$$-\frac{1}{2}\mu_o^2 \,(Tr\,(\rho^+\rho) + \nu_1 Tr\,(\rho^+\rho\rho^+\rho)$$
$$+ \nu_2 (Tr(\rho^+\rho))^2) \qquad (2.19)$$

where,

$$\rho \equiv \eta + \Sigma \; , \qquad (\rho = \rho^+)$$

and,

$$D_\mu \rho = \partial_\mu \rho - ig\,(\hat{Y}^L_\mu \rho - \rho \hat{Y}^R_\mu) - ig_o \frac{\lambda_o}{2}(Y^{L,o}_\mu - Y^{R,o}_\mu)\rho$$

$$L_{V,A} = -\frac{1}{4}\mathrm{Tr}\,(Y^L_{\mu\nu} Y^L_{\mu\nu} + Y^R_{\mu\nu} Y^R_{\mu\nu}) - \frac{1}{2} m_o^2 \,(\mathrm{Tr}\,(\hat{X}^L_\mu + C^L_\mu)^2 + \mathrm{Tr}\,(\hat{X}^R_\mu + C^R_\mu)^2) - \frac{1}{2} m_o^2 Y_\mu^{V_o} Y_\mu^{V_o} - \frac{1}{2}(m_o^2+m_A^2)(Y_\mu^{A_o} - \frac{\sqrt{3}}{4g_o}\mathrm{Tr}(\partial_\mu \phi))^2 \qquad (2.20)$$

where $Y^{L,R}_\mu$, $\hat{Y}^{L,R}_\mu$ and $Y^{L,R}_{\mu\nu}$ are given by expressions similar to those for $X^{L,R}_\mu$, $\hat{X}^{L,R}_\mu$ and $X^{L,R}_{\mu\nu}$ and*

$$C^L_\mu = \frac{1}{ig} S^+_L \partial_\mu S_L = \frac{1}{ig} e^{-i\phi} e^{-i\chi'} \partial_\mu (e^{i\chi} e^{i\phi})$$

$$C^R_\mu = \frac{1}{ig} S^+_R \partial_\mu S_R = \frac{1}{ig} e^{i\phi} e^{-i\chi} \partial_\mu (e^{i\chi} e^{-i\phi}) \qquad (2.21)$$

$$L_{non.min.} = ih_o \,\mathrm{Tr}\,(Y^L_{\mu\nu} D_\mu \rho D_\nu \rho^+ + Y^R_{\mu\nu} D_\mu \rho^+ D_\nu \rho) - \frac{h_1}{4}\mathrm{Tr}\,(Y^L_{\mu\nu} Y^L_{\mu\nu} \rho^+ \rho + Y^R_{\mu\nu} Y^R_{\mu\nu} \rho^+ \rho) - \frac{h_2}{4}\mathrm{Tr}\,(Y^L_{\mu\nu} \rho\, Y^R_{\mu\nu} \rho^+) \qquad (2.22)$$

Finally, the symmetry breaking term $L_{S.B.}$ becomes,

$$L_{S.B.} = \frac{\lambda}{2} \exp\,(2i\,\mathrm{Tr}\,(\phi)) \det \rho + h.c. + \frac{1}{2}\mathrm{Tr}\,(A\,(S_L \rho S_R^+ + S_R \rho^+ S_L^+)) \qquad (2.23)$$

---

* C^L and C^R are expressed as power series in χ and ϕ by making use of the identity

$$e^{-i\theta} \partial_\mu e^{+i\theta} = i[\partial_\mu \theta + \frac{(-i)}{2!}[\theta, \partial_\mu \theta] + \frac{(-i)^2}{3!}[\theta,[\theta, \partial_\mu \theta]] + .$$

and a similar relation for $e^{i\theta} \partial_\mu e^{i\theta}$.

The Lagrangian has linear terms in the fields σ_o and σ_8 coming from the polynomial in ρ fields in expression (2.19) and from the symmetry breaking term (2.23). Substituting $\rho = \eta + \Sigma$ in these terms and collecting linear terms in Σ fields we obtain,

$$
\begin{aligned}
& -\frac{1}{2}\mu_o^2 \Big[\mathrm{Tr}(\eta+\Sigma)^2 + \nu_1 \mathrm{Tr}(\eta+\Sigma)^4 \\
& + \nu_2 (\mathrm{Tr}(\eta+\Sigma)^2)^2\Big] + \frac{\lambda}{2}(\exp(2i\,\mathrm{Tr}(\phi)) + \text{h.c.}) \\
& \times \det(\eta+\Sigma) + \frac{1}{2}\mathrm{Tr}\Big[A\, S_L (\eta+\Sigma) S^+_R + \\
& + A\, S_R (\eta+\Sigma) S_L^+\Big] \\
& \quad = -\frac{1}{2}\mu_o^2 \Big[2\,\mathrm{Tr}(\eta\Sigma) + 4\nu_1 \mathrm{Tr}(\eta^3\Sigma) \\
& \quad + 4\nu_2 \mathrm{Tr}(\eta^2).\mathrm{Tr}(\eta\Sigma)\Big] + \lambda \det\ \mathrm{Tr}(\eta^{-1}\Sigma) \\
& \quad + \mathrm{Tr}(A\Sigma) + \ldots .
\end{aligned}
$$

In order that VEV's of σ_o and σ_8 fields may vanish, we must have, in tree approximation, the coefficients of σ_o and σ_8 equal to zero. This gives two equations which can be written as,

$$
\begin{aligned}
& \mu_o^2 (\eta + 2\nu_1 \eta^3 + 2\nu_2 \mathrm{Tr}(\eta^2)\eta) \\
& \quad = A + \lambda \eta^{-1} \det \eta
\end{aligned}
\tag{2.24}
$$

The vector and axial vector fields,

$$
\begin{aligned}
Y_\mu^{V_k} &= \frac{1}{2}(Y_\mu^{L,k} + Y_\mu^{R,k}) \\
Y_\mu^{A_k} &= \frac{1}{2}(Y_\mu^{L,k} - Y_\mu^{R,k})
\end{aligned}
$$

have mixings with the scalar and pseudoscalar fields coming from the terms of the type $Y_\mu^V \cdot \partial_\mu \sigma$ and $Y_\mu^A \cdot \partial_\mu \phi$. To remove these mixings we write,

$$Y_\mu^{V_k} = y_\mu^{V_k} - C_{kl}^V \quad \partial_\mu \sigma_l \tag{2.25}$$

and $$Y_\mu^{A_k} = y_\mu^{A_k} - C_{kl}^A \quad \partial_\mu \phi_l$$

and determine the C's from the requirement that there be no mixing of type $y_\mu^V \cdot \partial_\mu \sigma$ and $y_\mu^A \cdot \partial_\mu \phi$. We now define renormalized vector and axial vector fields by the equations,

$$V_\mu^k = Z_{V_k}^{-1/2} \; y_\mu^{V_k} \quad , \quad k = 1,\ldots, 7$$
$$A_\mu^k = Z_{A_k}^{-1/2} \; y_\mu^{A_k} \quad , \quad k = 1,\ldots, 7 \tag{2.26}$$

and determine the Z'S from the requirement that the fields V_μ and A_μ have standard kinetic energy coefficients. We obtain, using particle symbols as subscripts,

$$Z_\rho^{-1} = 1 + (h_1 + h_2)\; \alpha^2$$
$$Z_{K^*}^{-1} = 1 + h_1\; (\alpha^2 + \gamma^2)/2 + h_2\; \alpha\gamma$$
$$Z_{A_1}^{-1} = 1 + (h_1 - h_2)\; \alpha^2$$
$$Z_{K_A}^{-1} = 1 + h_1\; (\alpha^2 + \gamma^2)/2 - h_2\; \alpha\gamma$$

and the squared masses of these particles are ,

$$m_\rho^2 = m_o^2 Z_\rho$$

$$m_{K^*}^2 = [m_o^2 + g^2 (\alpha - \gamma)^2/2] Z_{K^*}$$

$$m_{A_1}^2 = (m_o^2 + 2g^2 \alpha^2) Z_{A_1}$$

$$m_{K_A}^2 = [m_o^2 + g^2(\alpha + \gamma)^2/2] Z_{K_A}$$

If the bilinear terms in the 8th and 0th vector and axial vector fields are written as,

$$\sum_{i,j=0,8} \Big[-\frac{1}{4} (\partial_\mu V_\nu^i - \partial_\nu V_\mu^i) (K_V)_{ij} (\partial_\mu V_\nu^j - \partial_\nu V_\mu^j) - \frac{1}{2} V_\mu^i (M_V^2)_{ij} V_\mu^j + V \to A \Big]$$

we have:

$$(K_V)_{88} = 1 + (h_1 + h_2)(\alpha^2 + 2\gamma^2)/3$$

$$(K_V)_{00} = 1 + (h_1 + h_2)(2\alpha^2 + \gamma^2)/3$$

$$(K_V)_{08} = (K_V)_{80} = \frac{\sqrt{2}}{3} (h_1 + h_2)(\alpha^2 - \gamma^2)$$

$$(M_V^2)_{ij} = m_o^2 \delta_{ij}$$

$$(K_A)_{88} = 1 + (h_1 - h_2)(\alpha^2 + 2\gamma^2)/3$$

$$(K_A)_{00} = 1 + (h_1 - h_2)(2\alpha^2 + \gamma^2)/3$$

$$(K_A)_{08} = (K_A)_{80} = \frac{\sqrt{2}}{3} (h_1 - h_2)(\alpha^2 - \gamma^2)/3$$

$$(M_A^2)_{88} = m_o^2 + 2g^2 (\alpha^2 + 2\gamma^2)/3$$

$$(M_A^2)_{00} = m_o^2 \, (1 + m_A^2/m_o^2) + 2 \, g_o^2 \, (2\alpha^2 + \gamma^2)/3$$

$$(M_A^2)_{80} = (M_A^2)_{08} = \frac{\sqrt{2}}{3} \, g g_o \, (\alpha^2 - \gamma^2) \qquad (2.27)$$

The mixing problem for various multiplets is discussed in the Appendix A. For the vector mesons the renormalized masses are,

$$m_\omega^2 = m_o^2 \,/\, (1 + (h_1 + h_2) \, \alpha^2), \quad m_\phi^2 = m_o^2 / \, (1 + (h_1 + h_2) \, \gamma^2)$$

and the mixing angle has the 'nonet' value

$$\tan 2\theta = 2\sqrt{2}, \qquad \theta = 35.5^\circ$$

The renormalized scalar and pseudoscalar fields are defined by equations similar to Eq. (2.26). The renormalization constants and masses for the pseudoscalar and scalar mesons are,

$$Z_\pi = 1 + 2g^2 \, \alpha^2 \,/\, m_o^2$$

$$Z_K = 1 + g^2 \, (\alpha + \gamma)^2 \,/\, 2m_o^2$$

$$m_\pi^2 = Z_\pi \, (a/\alpha)$$

$$m_K^2 = Z_K \, (a+c)/(\alpha + \gamma)$$

$$Z_{S_\pi} = 1 \quad , \quad Z_{S_K} = 1 + g^2 \, (c - \gamma)^2/2 \, m_o^2.$$

$$m_{S_\pi}^2 = \mu_o^2 \, (\nu_o - 12 \, \nu_1 \, \alpha^2) - \lambda \, \alpha$$

$$m_{S_K}^2 = Z_{S_K} \, (a-c)/(\alpha - \gamma).$$

The mass squared matrix for the 8th and 9th pseudoscalar mesons is

$$(M_P^2)_{88} = 4(a\alpha + 2c\gamma)/3 \, f_{\phi_8}^2$$

$$(M_P^2)_{00} = 4(2a\alpha + c\gamma)/3\, f_{\phi_o}^2 + 3\lambda\, \alpha/f_o^2$$

$$(M_P^2)_{08} = 8(a\alpha - c\gamma)/(\sqrt{18}\, f_{\phi_8}\, f_{\phi_o})$$

The kinetic energy matrix for these mesons is,

$$K = K' - \xi^T (M_A^{-2})\, \xi \tag{2.28}$$

where,

$$(K)_{88} = 2m_o^2/g^2\, f_{\phi_8}^2 \,, \quad (K)_{00} = 2(m_o^2 + m_A^2)/g_o^2\, f_{\phi_o}^2 \,,$$

$$(K)_{08} = (K)_{80} = 0,$$

$$(\xi)_{88} = -\sqrt{2}\, m_o^2/gf_{\phi_8} \,, \quad (\xi)_{00} = -\sqrt{2}(m_o^2 + m_A^2)/g_o\, f_{\phi_o}$$

$$(\xi)_{08} = (\xi)_{80} = 0$$

where f_{ϕ_8} and f_{ϕ_o} are given by Eqs. (2.16) and M_A^{-2} is inverse of the unrenormalized mass squared matrix for 8th and 0th axial vector mesons (see Eq. (2.27)).

2.3 Determination of Parameters:

Most of the parameters were determined through experimental masses. The crucial parameter $\delta \equiv \alpha/\gamma$, characterizing the relative strengths of breaking of SU(3) and of SU(3) X SU(3) by vacuum was varied in a suitable range. For each value of δ the nine parameters $\nu_o\, \mu_o^2$, $\lambda\gamma$, ν_1/ν_o, m_o, $g\,\gamma$, g_o/g, $h_1\gamma^2$, $h_2\gamma^2$ and m_A/m_o were fixed by a least square fit to the twelve masses of the vector, axial vector and the pseudoscalar mesons[26].

Then m_{S_π} and m_{S_K} were calculated and were found to be sensitive to value of δ. For $\delta = 0.84$ best overall fit was obtained for the mesonic masses. The values of various parameters for $\delta = 0.84$ are,

$$\nu_o \mu_o^2 = -4.64\ m_\pi^2 \ , \ \lambda\gamma = 8.84\ m_\pi^2 \ , \ \nu_1/\nu_o = 1.06 \ ,$$

$$m_o = 0.668\ m_\rho \ , \ g\gamma = 0.703\ m_\rho \ , \ g_o/g = -0.95 \ ,$$

$$h_1\gamma^2 = -0.658 \ , \ h_2\gamma^2 = -0.058 \ , \ m_A/m_o = 0.99 \ .$$

The calculated and experimental values[27] of masses of the vector axial vector and pseudoscalar mesons are given in Table I. The masses of S_π and S_K are predicted to be 949 MeV and 1025 MeV respectively. We identify these particles with the resonances π_N (980) and K_N(1080). The masses of S_η and S_η' depend on an additional parameter ν_2 and we list below values of m_{S_η} and $m_{S_\eta'}$ for some values of ν_2.

ν_2/ν_o	m_{S_η}	$m_{S'_\eta}$
− 0.2	1054	374
− 0.1	1060	561
0.0	1068	694
0.1	1083	798
0.2	1107	878

For rest of the discussion we assume $\nu_2 = 0$ and identify S_η and S_η' with S*(1060) and ϵ(700) resonances respectively.

Table I: Meson masses in MeV.

	ρ	K^*	ω	ϕ	A_1	K_A	D	E	π	K	η	X^0
Experimental value	765	891.7	783.9	1019.5	1070	1242	1286	1422	140	494	548.8	957.3
Calculated Value	776	879	776	1024	1085	1244	1281	1424	140	498	543	958

The values of the nine parameters given in Eq. (2.29) were obtained by a least square fit to the above twelve masses. The values in the second row were used in getting the best fit. The third row contains final calculated values (Ref. 27).

The parameters $\epsilon = a_8/a_0$ and $\xi = n_8/n_0$ which characterize the relative strengths of the breaking of the SU(3) and SU(3) X SU(3) symmetry by the vacuum and by the Lagrangian are given by,

$$\epsilon = \sqrt{2}\,(a - c)/(2a+c) = -1.25$$

$$\xi = \sqrt{2}\,(\alpha - \gamma)/(2\alpha+\gamma) = -0.084$$

The value of ϵ is same as obtained by Gell Mann Oakes and Renner[15], and is close to the SU(2) X SU(2) value $-\sqrt{2}$ The value of ξ is close to the SU(3) value 0 Thus for vacuum SU(3) is an approximate symmetry while for the Lagrangian SU(2) X SU(2) is a better symmetry as compared to SU(3). These conclusions are in agreement with recent studies of chiral symmetry breaking using Ward identities[28] and other Lagrangian models [29]

CHAPTER III

LOW ENERGY MESONIC INTERACTIONS

The effective Lagrangian obtained in the previous chapter is used to study the strong interactions of mesons. We first consider the meson decays with two and three particle final state. The low energy parameters for the pseudoscalar meson-meson scattering are calculated. All the above mentioned calculations are done in tree approximation only

3.1 Two Particle Decays of Mesons.

For the decay process,

$$A \rightarrow B + C$$

the width is given by,

$$\Gamma(A \rightarrow B + C) = \frac{k_{c.m.}}{8\pi\, M_A^2} \frac{1}{(2J_A+1)} \sum_{spins} |m|^2 \tag{3.1}$$

where M_A and J_A are mass and spin of the decaying particle and $k_{c.m.}$ is the center of mass momentum of the final particles and is given by,

$$k_{c.m}^2 = \left[M_A^2 - (M_B + M_C)^2 \right] \left[M_A^2 - (M_B - M_C)^2 \right] / \, 4M_A^2 \tag{3.2}$$

and m is the S-matrix element defined by,

$$\langle\, BC \,|\, S \,|\, A \,\rangle = i(2\pi)^4\, \delta(p_A - p_B - p_C) \frac{1}{(2\pi)^{9/2}} \frac{m}{\sqrt{8\omega_A\, \omega_B \omega_C}} \tag{3.3}$$

There are two more parameters g and h_0 to be fixed before we can calculate the rates for two body decays of mesons The value of g is fixed using the π_{l_2} and the K_{l_2} decay rates in the following way.

The ratio F_K/F_π does not involve any new parameter and is predicted to be 1.04 (see Section 4.2), for $\delta = 0.84$. The rates $\Gamma(\pi \to \mu\nu)$ and $\Gamma(K \to \mu\nu)$ are given by,

$$\Gamma(\pi \to \mu\nu) = \frac{G^2}{8\pi^2} (F_\pi \cos\theta_A)^2 \; M_\pi m_\mu^2 (1 - m_\mu^2 / M_\pi^2)$$

$$\Gamma(K \to \mu\nu) = \frac{G^2}{8\pi^2} (F_K \sin\theta_A)^2 \; M_K m_\mu^2 (1 - m_\mu^2 / M_K^2)$$

Using the experimental values[27],

$$\tau(\pi \to \mu\nu) = 2.603 \times 10^{-8} \text{ sec}^{-1}$$

$$\tau(K \to \mu\nu) = 1.937 \times 10^{-8} \text{ sec}^{-1}$$

for the lifetimes we obtain,

$$\left(\frac{F_K}{F_\pi} \tan\theta_A \right)^2 = 0 \; 775$$

From above Eq. and $F_K/F_\pi = 1.04$, we get,

$$\tan\theta_A = 0.26$$

Using this value for the Cabibbo angle θ_A, we obtain $F_\pi = 93.3$ MeV from $\Gamma(\pi \to \mu\nu)$. The parameter $g\alpha$ and Z_π are already known and α ($= F_\pi Z_\pi^{1/2} / \sqrt{2}$) is now known. This gives

$$g^2/4\pi = 1.53.$$

Using this value of g we can calculate the widths for the $V \to PP$ and $A \to VP$ decays and are given (in MeV), as function of $h(= - m_o^2 h_o/g)$ by,

$$\Gamma(\rho \to 2\pi) = 84.0\ (1+2.47h + 1.49h^2)$$

$$\Gamma(K^* \to K\pi) = 26\ 7\ (1+3\ 07h + 2.32h^2)$$

$$\Gamma(\phi \to K\bar{K}) = 1.37\ (1+1.94h + 1.77h^2)$$

$$\Gamma(A_1 \to \rho\pi) = 123.0(1-4.14h + 4.20h^2)$$

$$\Gamma(K_A \to K^*\pi) = 79.4\ (1-4.82h + 6\ 11h^2)$$

$$\Gamma(E \to K^*\bar{K} + \bar{K}^*K) = 64.7(1-7.67h+14.7h^2)$$

These rates are given in Table II for some values of h. Overall good agreement with experiments is obtained for h = 0.1. The width for $K^* \to K\pi$ is, however, small in the range 0.02 – 0.18 for h.

Widths for allowed two particle decay modes $A \to PS$ and $S \to PP$ are calculated for h = 0.1 and are given in Table III. In the case of axial vector mesons A_1 and K_A widths for two body decays are not known at present. The total widths for three particle decays $A_1 \to 3\pi$ and $K_A \to K\pi\pi$ will be calculated in the next section and will be compared with experiments $\Gamma(E \to \pi_N\pi)$ is to be compared with experimental value 34.5 ± 5.

Experimentally $\Gamma(\pi_N \to \eta\pi) = 50 \pm 10$. For other scalar mesons the widths are not known accurately. Total widths of S_K and ε are very large. For $S^*(1060)$ only the total width is

Table II $V \to PP$ and $A \to VP$ Decay Widths (in MeV) for Some Values of h

h	0.02	0.04	0.06	0.08	0.10	0 12	0 14	0.16	0.18	Experimental value[a]
$\rho \to 2\pi$	88.1	92.4	96 7	101.2	105.8	110 5	115 3	120.3	125.3	125±20
$K^* \to K\pi$	29.2	30.9	32.7	34.6	36.5	38.4	40.5	42.5	44.6	50
$\phi \to K\bar{K}$	2.2	2.4	2.6	2.8	2.9	3.1	3.4	3.6	3.8	3.2
$A_1 \to \rho\pi$	112.9	103.4	94.4	85.8	77.6	69.8	62.8	55.6	49.1	95±35
$K_A \to K^*\pi$	71.4	63.9	56.8	50.1	43.9	38.1	32.8	27.9	23.4	40–130
$E \to (K^*\bar{K}+\bar{K}^*K)$	57.6	48.4	40.0	32.5	25.7	19.7	14.5	10.1	6.5	35 5

a See Ref. 27.

Table III: S → PP and A → SP Decays.

Decay	Width (MeV)
$\pi_N \rightarrow \eta\pi$	31.5
$S_K \rightarrow K\pi$	94.4
$\epsilon \rightarrow \pi\pi$	38.6
$S^* \rightarrow \pi\pi$	54.7
$S^* \rightarrow K\bar{K}$	26.7
$A_1 \rightarrow \epsilon\pi$	28.5
$K_A \rightarrow S_K\pi$	33.5
$K_A \rightarrow \epsilon K$	4.8
$D \rightarrow \pi_N\pi$	8.5
$E \rightarrow \pi_N\pi$	44.0
$D \rightarrow \epsilon\eta$	6.1
$E \rightarrow \epsilon\eta$	17.4

known to be 60 $\pm$ 20 and the branching ratio $\Gamma(S^* \to K\bar{K})/\Gamma(S^* \to \pi\pi)$ is not known

Parametrizing the $A_1 \to \rho\pi$ matrix element as,

$$m \quad (A_1(Q) \to \quad (k) + \pi(p)) = g_s \, \mathcal{E}_A \, \mathcal{E}_\rho + g_d(\mathcal{E}_A . k)(\mathcal{E}_\rho . Q) \quad (3.4)$$

we have,

$$\begin{aligned} g_s &= \frac{1}{2}(m_\rho^2 - m_{A_1}^2 - m_\pi^2) \, g_1^{A_1\rho\pi} - \frac{1}{2}(m_{A_1}^2 - m_\rho^2 - m_\pi^2) g_2^{A_1\rho\pi} \\ g_d &= (g_2^{A_1\rho\pi} - g_1^{A_1\rho\pi}) \end{aligned} \quad (3.5)$$

where the coupling constants $g_1^{A_1\rho\pi}$ and $g_2^{A_1\rho\pi}$ are defined in the Appendix B

Numerically we have $G_{A_1\rho\pi} = m_{A_1}^2 \, g_d/g_s = -1.66$. This is to be compared with the value -2 obtained from field algebra and KSRF relation.[30] In our model the KSRF relation does not hold (see Section 4.2) and this may be the cause of the deviation in the value of $G_{A_1\rho\pi}$ from the field algebra value. In the linear Lagrangian model of Bhargava[29] $G_{A_1\rho\pi} = -1$ which means a larger value for g_s. This may be due to an extra coupling term $\vec{A}_\mu \quad \vec{\rho}_\mu \times \vec{\pi}$ contributing to g_s which comes from $-\frac{1}{2}\,\mathrm{Tr}\,(D_\mu M^+, D_\mu M)$ term in L_M and is absent in our model Most of the theoretical studies of $A_1 \, \rho\pi$ system favour a dominant s-wave decay[31] Some theoretical models favour a predominant d-wave decay.[32] The $\rho - \pi$ angular distribution measurements[33] give,

$$\frac{m_\rho}{m_{A_1}} \, |g_T/g_L| \;=\; 0.48 \pm 0.12 \tag{3 6}$$

where, g_T and g_L are related to g_s and g_d by,

$$g_s \;=\; k^2 \, g_T \quad , \quad g_d \;=\; g_L - E_\rho \;\; g_T/m_{A_1}$$

To satisfy (3.6) $G_{A_1 \, \rho\pi}$ must obey either of the two inequalities

$$-0.05 \leq G_{A_1 \, \rho\pi} \leq -0\ 03$$

$$0.08 \leq G_{A_1 \, \rho\pi} \leq 0\ 22$$

These results are compatible neither with dominant s-wave nor with dominant d-wave decays. Some attempts[34] have been made to understand these results by modifying the hard pion approach.

For $h_1 \;=\; h_2$, $h \;=\; 0$ we get,

$$g_s \;=\; m_\rho^2 \;/F_\pi \quad , \; g_d \;=\; 0 \tag{3.7}$$

if we use KSRF relation Equations (3.7) are the soft pion results of Renner[35] and of Geffen[36]. The soft pion values for g_s and g_d predict a large (600 MeV) width for A_1-decay. Improved results have been obtained by using Ward identities for three point vertices[37]. In the dispersion theoretic formalism the soft pion result follows if all form factors are assumed to obey unsubtracted dispersion relations. This assumption has been criticized by a number of authors who obtain the improved results by using once subtracted dispersion relations for some of the form factors.[38]

3.2 Decays with Three Particle Final State

We now consider the three body decays of type,

$$A(Q) \rightarrow B(q_1) + C(q_2) + D(q_3)$$

Let,

$$\langle BCD \mid S \mid A \rangle = i(2\pi)^4 \delta^4 (Q - q_1 - q_2 - q_3) \frac{1}{(2\pi)^6}$$

$$\times \frac{m}{\sqrt{16\omega_Q \, \omega_1 \, \omega_2 \, \omega_3}}$$

The width is, then given by[39],

$$\Gamma(A \rightarrow B+C+D) = \frac{1}{64\pi^3 M_A} \int_{M_A}^{\omega_{max}} d\omega_2 \int_{\omega_-}^{\omega_+} \frac{1}{(2J_A+1)} \sum_{spins} |m|^2 d\omega_1 \tag{3 8}$$

where,

$$\omega_{max} = \frac{M_A^2 + M_2^2 - (M_1 + M_3)^2}{2 M_A}$$

and $\omega_{\pm}$ are the two roots of the quadratic equation

$$x\,\omega^2 + y\,\omega + z = 0$$

where,

$$x = 4((\omega_2 - M_A)^2 - \omega_2^2 + M_2^2)$$

$$y = 4t\,(\omega_2 - M_A)$$

$$z = t^2 + 4M_1^2\,(\omega_2^2 - M_2^2)$$

and $$t = M_A^2 - M_3^2 + M_1^2 + M_2^2 - 2M_A\,\omega_2.$$

We calculate the widths for the following processes (1) $A_1 \to 3\pi$, (2) $K_A \to K\pi\pi$, (3) (D, E) $\to K\bar{K}\pi$; (4) (D,E) $\to \eta\pi\pi$ and (5) $\eta' \to \eta\pi\pi$. The graphs contributing to these processes are shown in Fig.1. While calculating the decay widths it was noted that some of the propagators, like ρ. K' etc., become singular in the range of integration. In order to avoid these singularities the finite width approximation was made and in propagators mass M of the intermediate particle was replaced by $M - i\,\Gamma/2$ where Γ is the width of the intermediate particle.

The predicted values for three particle decay widths are given in Table IV For the $A_1 \to 3\pi$ decays maximum contribution comes from the $\rho\pi$ intermediate state. The scalar meson and the direct contributions are found to be small. This is in contrast with some models where the direct contributions are found to be small[40]. For the $K_A \to K\pi\pi$ and also for (D,E) $\to K\bar{K}\pi$ the K^* graph contributions are large and the scalar mesons contributions to amplitude for these processes are small. For the $E \to \eta\pi\pi$ decay the $\pi_N\pi$ intermediate state contributions are large as compared to that for S_η and S_η' states. These results are inagreement with results of Bhargava[29] except that in his work $S_K\,\pi$ contributes significantly to K_A width and there is large interference of the $S_K\,\pi$ and $K^*\pi$ intermediate state contributions.

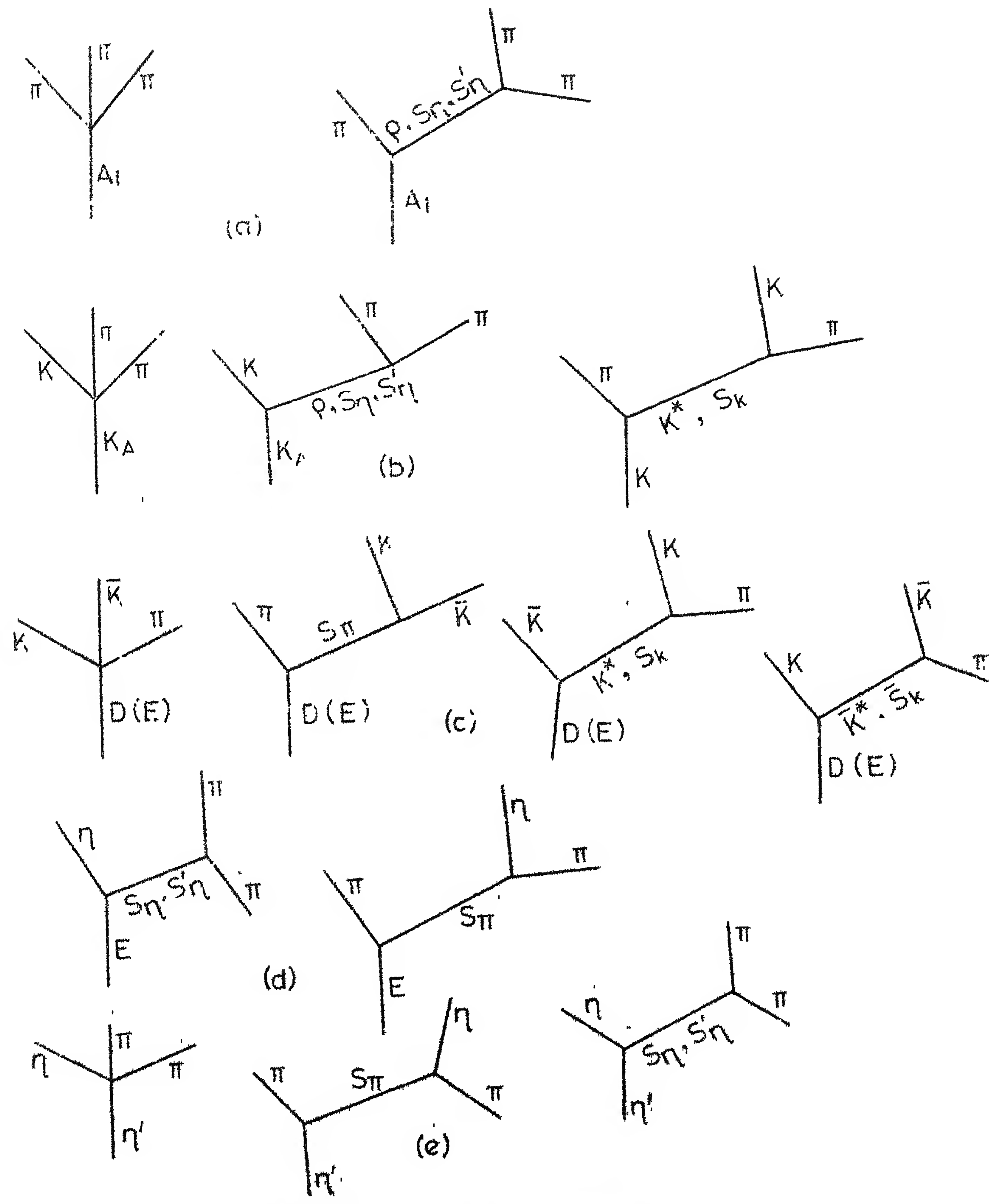


Fig.1 Diagrams for three particle decays of mesons (a) $A_1 \longrightarrow 3\pi$ (b) $K_A \rightarrow K\pi\pi$ (c) $D(E) \rightarrow K\bar{K}\pi$ (d) $E \rightarrow \eta\pi\pi$ (e) $\eta' \longrightarrow \eta\pi\pi$

Table IV Rates (in MeV) for Three Body Decay Modes of Axial Vector Mesons and the X^o.

Decay	Width	With in the[a] limit $(\mu_o^2) \to \infty$	Experimental value
$A_1 \to 3\pi$	67.5	51.5	95 ± 35
$K_A \to K\pi\pi$	52.4	49.0	40 - 130
$E \to K\bar{K}\pi$	19 5	18.5	69 ± 4[b]
$E \to \eta\pi\pi$	30.7	26.5	
$D \to K\bar{K}\pi$	0.24	0.6	33 ± 4[b]
$D \to \eta\pi\pi$	7.6	7.1	
$X^o \to \eta\pi\pi$	0.013	0.010	4

a See Section 3.4.

b The value quoted are the total widths for the decaying meson and the two modes are the important ones contributing to the total width.

3.3 Meson Meson Scattering

The amplitude T_{abcd} (s,t,u) for the scattering of two pseudoscalar mesons[41]

$$P_a(q_a) + P_b(q_b) \rightarrow P_c(q_c) + P_d(q_d)$$

is defined as

$$\langle P_c\, P_d \,|\, S \,|\, P_a\, P_b \rangle = \delta_{fi} + i(2\pi)^4\, \delta(q_1 + q_b - q_c - q_d)$$

$$\times \frac{T_{abcd}\,(s,t,u)}{(2\pi)^6 \sqrt{16\, \omega_a\, \omega_b\, \omega_c\, \omega_d}}$$

where the Mandelstam variables are,

$$s = -(q_a + q_b)^2$$

$$t = -(q_a - q_c)^2$$

$$u = -(q_a - q_b)^2$$

The scattering amplitude F, related to the differential cross section by,

$$\frac{d\sigma}{d\Omega} = |\, F \,|^2$$

is,

$$F = \frac{1}{8\pi\sqrt{s}}\, T(s,t,u)$$

The partial wave expansion of the amplitude is,

$$F(|\vec{q}|^2, \cos\theta) = \sum_{l=0}^{\infty} (2l+1)\, P_l\, (\cos\theta)\, f_l\, (|\vec{q}|^2)$$

where $\vec{q}^2$ and θ are square of center of mass momentum and the scattering angle and are related to the Mandebtam variables by,

$$s = (\omega_a + \omega_b)^2$$

$$t = (\omega_a - \omega_b)^2 - 2\,\vec{q}^2\,(1 + \cos\theta)$$

$$u = -\,|\vec{q}|^2\,(1 - \cos\theta)$$

where,

$$\omega_a^2 = |\vec{q}|^2 + M_a^2 \quad \text{etc}$$

The scattering length, a_l, and effective range, r_l, for the l-th partial wave are defined by,

$$q^{2l}\ \mathrm{Re}\,(f_l\,(|\vec{q}|^2))^{-1} = a_l^{-1} + \frac{1}{2}\,r_l\,|\vec{q}|^2 + O\,(|\vec{q}|^4)$$

(i) $\pi - \pi$ Scattering.

The amplitude $T_{abcd}\,(s,t,u)$ for the process,

$$\pi_a + \pi_b \to \pi_c + \pi_d$$

has the isospin structure

$$T_{abcd}\,(s,t,u) = \delta_{ab}\,\delta_{cd}\,A(s,t,u) + \delta_{ac}\,\delta_{bd}\,B(s,t,u) + \delta_{ad}\,\delta_{bc}\,C(s,t,u),$$

and the amplitudes T^I $(I = 1,2,3)$ corresponding to the isospin channel I are,

$$T^0 = 3A + B + C$$

$$T^1 = B - C$$

$$T^2 = B + C$$

We calculate the $\pi - \pi$ scattering amplitude in the tree approximation. The graphs which contribute to the process are shown in Fig. 2(a).

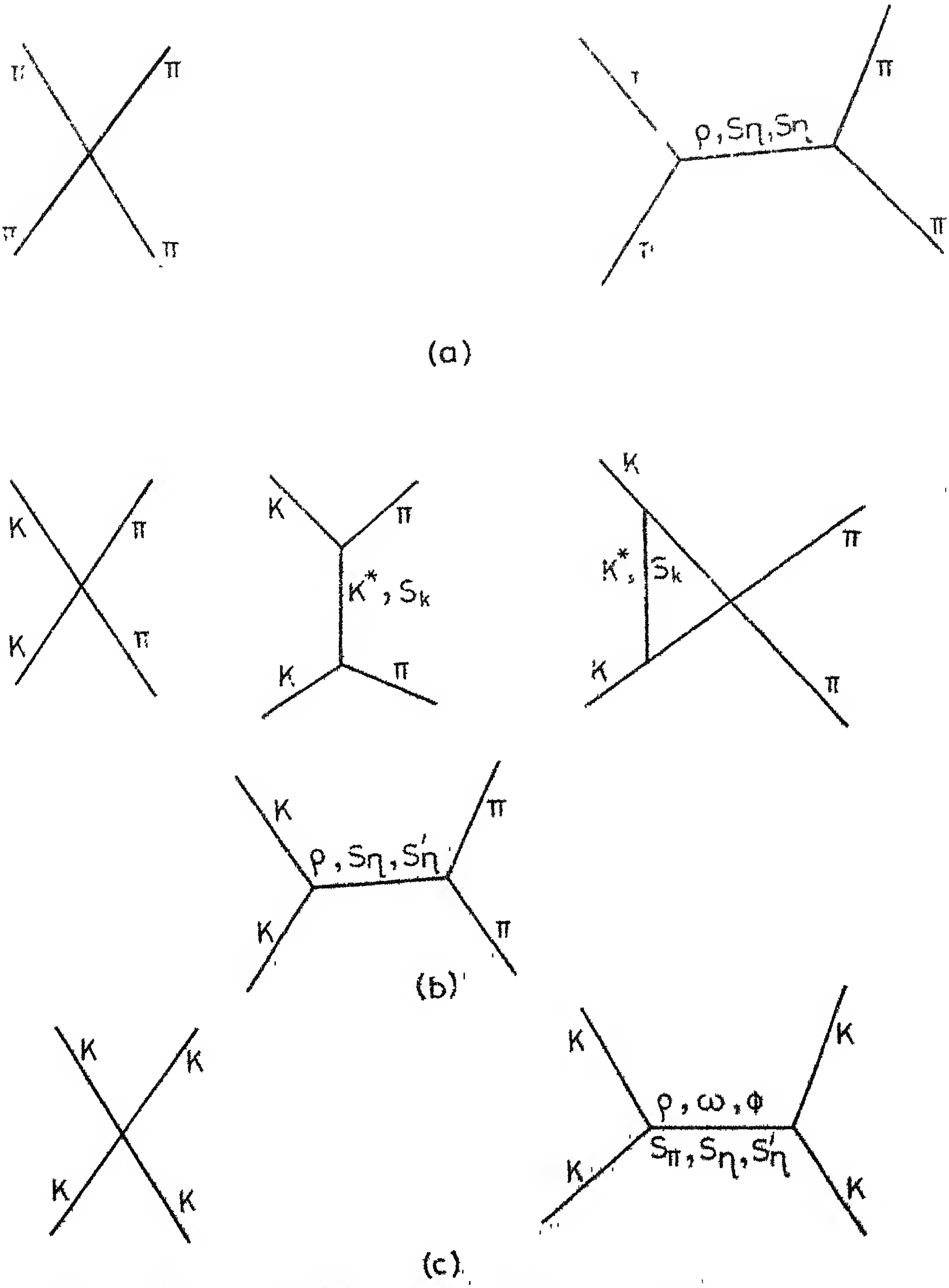


Fig. 2 Tree graphs contributing to peudoscalar meson scattering processes (a) π−π scattering (b) K−π scattering (c) K−K scattering.

The amplitudes $B(s,t,u)$ and $C(s,t,u)$ are related to $A(s,t,u)$ by crossing symmetry,

$$B(s,t,u) = A(t,s,u)$$
$$C(s,t,u) = A(u,t,s)$$

and $A(s,t,u)$ is given by[42],

$$\begin{aligned}
A(s,t,u) = & - g^{(1)}_{\pi\pi}(-2(s-2m^2_\pi)^2 + (t-2m^2_\pi)^2 + (u-2m^2_\pi)^2) \\
& - 2g^{(2)}_{\pi\pi}(4m^2_\pi - 3s) + 8g^{(3)}_{\pi\pi} \\
& + \frac{(g^{(1)}_{\rho\pi\pi} - g^{(2)}_{\rho\pi\pi}\, u)(s-t)}{m^2_\rho - u} + \frac{(g^{(1)}_{\rho\pi\pi} - g^{(2)}_{\rho\pi\pi}\, t)(s-u)}{m^2_\rho - t} \\
& + \left[\frac{(2g^{(1)}_{S_\eta\pi\pi}\, m^2_\pi - 2g^{(2)}_{S_\eta\pi\pi} - g^{(1)}_{S_\eta\pi\pi}\, s)^2}{m^2_{S_\eta} - s} + S_\eta \to S'_\eta\right]
\end{aligned}$$

The s-wave scattering lengths are given by,

$$32\,\pi\, m_\pi \quad a^0_s = (7m^2_\pi/F^2_\pi) + .$$

and

$$32\pi\, m_\pi \quad a^2_s = -(2m^2_\pi/F^2_\pi) + . \; .$$

where the dots stand for the additional contributions coming from S_η and S'_η exchanges and the terms written explicitly are the contributions of the contact term and the ρ-meson exchange. Neglecting the contributions of scalar mesons, which are less than a few percent of the total value, our results

agree with soft pion calculations of Weinberg.[43] In case of p-wave scattering we have,

$$24 \pi m_\pi a_p^1 = F_\pi^{-2} + O\left(\frac{m_\pi^2}{m_\rho^2}\right) .$$

The leading term in the expansion of right hand side in powers of m_π/m_ρ agrees with soft pion result

Similar observations have been made by A.K. Bhargava[29] who calculated the scattering lengths in a linear phenomenological Lagrangian model but our results are different in the following two respects Firstly the contributions of the scalar mesons to the scattering lengths are small and insensitive to variations in ν_2 in contrast to results of Ref. 29, where the scalar meson contribution was found to be large and sensitive to variations in ν_2. The second difference being that we donot have to assume KSRF condition[44] to get current algebra results as was needed in Ref. 29. The numerical values of the scattering lengths and effective ranges are given in Table V. As the scalar meson contributions are negligible the predicted values are close to current algebra results

(ii) <u>K-π Scattering</u>

The isospin structure of the amplitude $T_{ia,jb}$ for the process,

$$K_i + \pi_a \rightarrow K_j + \pi_b$$

Table V $\pi - \pi$ Scattering Parameters. The s-wave scattering lengths and effective ranges are in units of m_π^{-1}. The p-wave scattering lengths and effective ranges are in units of m_π^{-3} and m_π respectively.

	a_s^0	r_s^0	a_s^2	r_s^2	a_p^1	r_p^1
Present model	0.16	-7.6	-0.045	64.3	0.035	21.0
Soft pion results	0.17±0.02	-7.3±0.7	-0 05±0.005	6.0±0.6	0 033±0 003	-
$\pi N \to 2\pi N$[a]	0 19±0.04	-0.059±0 015	-	-	0.038	-
$\pi N \to 2\pi N$[b]	$0.2^{+0.08}_{-0.1}$	$-5.2^{+1.5}_{-2.0}$	-	-	-	-

a From high energy peripheral reaction data (Ref. 45)

b From low energy data (see Ref. 46).

is given by,

$$T_{ia,\,jb} = \chi_j^+ \chi_i \delta_{ab} T_A + \chi_j^+ [\tau_b, \tau_a] \chi_i T_B$$

where χ_i and χ_j are isospin wave functions for the initial and final kaons respectively.

The amplitudes for the I = 3/2 and I = 1/2 channels are,

$$T^{3/2} = T_A - T_B$$

$$T^{1/2} = T_A + 2T_B$$

Figure 2(b) shows the graphs which contribute to K-π scattering in the tree approximation. As in the case of π-π scattering we find that the S_η and S_η' exchange, contributions are small and insensitive to variations in ν_2 The numerical values for the s-wave and the p-wave scattering parameters are given in Table VI and are compared with soft pion results of Weinberg[43] and the hard pion results of Paul Pond.[47]

(iii) <u>K-K Scattering</u>

The amplitude T_{ijkl} for the K-K scattering,

$$K_i + K_j \to K_k + K_l$$

has the form

$$T_{ijkl} = \delta_{ik}\delta_{jl} T_A + \delta_{il} \delta_{jk} T_B$$

and the definite isospin amplitudes are

$$T^0 = T_A - T_B$$

$$T^1 = T_A + T_B$$

Table VI K – π Scattering Parameters The s-wave parameter are in units of m_π^{-1} and p-wave scattering lengths and effective ranges are in units of m_π^{-3} and m_π respectively.

	$a_s^{1/2}$	$r_s^{1/2}$	$a_s^{3/2}$	$r_s^{3/2}$	$a_p^{1/2}$	$r_p^{1/2}$	$a_p^{3/2}$	$r_p^{3/2}$
Present model	0.16	-15	-0.058	70	0.014	-33	0.0033	170
Soft Pion calculation[a]	0.16 ±0.02	-5.0 ±0.5	-0.078 ±0.008	17±2	0.014 ±0.001	-	-	-
Hard Pion calculation[b]	0 115	-12.0	-0.090	14.2	0.0153	-35.5	0.005	-35.0

a Ref. 43

b The values quoted here are for F_K/F_π = 1.169 (Ref. 47).

The graphs contributing to K-K scattering are shown in Fig. 2(c)

The s-wave I = 1 and the p-wave I = 0 scattering lengths and effective ranges are predicted to be,

$$a_s^1 = -0\ 096\ m_\pi^{-1}\ , \qquad r_s^1 = -0.65\ m_\pi^{-1}$$

$$a_p^o = 0\ 0044\ m_\pi^{-3}\ , \qquad r_p^1 = 2.7\ m_\pi$$

The scattering lengths are close to the values obtained in other effective Lagrangian models[48] No estimates for effective ranges are available.

3.4 <u>Meson Decays and Scattering Parameters in a Limit</u>

If the limit $|\mu_o^2| \to \infty$ and $(\nu_o - 4\nu_1 \eta^3 - 4\ \nu_2\ \mathrm{Tr}(\eta^2)\eta) \to 0$ taken in such a way that Eq. (2.24) is satisfied the masses of the Chiral partners of the 'would be' Goldstone bosons tend to infinity and so do their couplings among themselves. The masses of the spin one mesons, the pseudoscalar mesons and of S_K and their couplings with S_π, S_η and S'_η remain unchanged. The processes in which the scalar mesons S_π, S_η and S'_η appear in the final state are not allowed. The contributions of these scalars to the processes, where they appear in the intermediate state only, vanishes in the limit.

The three body decay widths for $A \to 3P$ and $X^o \to \eta\pi\pi$ change slightly and are given in Table IV. The meson-meson scattering parameters do not change significantly in the above limit.

CHAPTER IV

CURRENTS AND MESON DECAY FORM FACTORS

In this chapter we obtain the vector and axial vector currents Currents are used to calculate the decay constants, F_π, F_K, F_κ for the scalar and pseudoscalar meson weak decays and f_ρ, f_ω f_ϕ for the neutral vector meson decays into lepton pairs l^+l^- The K_{l_3} form factors are calculated and discussed.

4.1 Currents

The infinitesimal variations (in the matrix form) of the fields under a general transformation are given by,

$$
\begin{aligned}
\delta M &= i\left(\frac{\epsilon^L . \lambda}{2}\right) M - iM\left(\frac{\epsilon^R \lambda}{2}\right) + i\,\frac{\epsilon_o^L \lambda_o}{2} M - iM\,\frac{\epsilon_o^R\,\lambda_o}{2} \\
\delta \hat{X}_\mu^L &= i\left[\frac{\epsilon^L\,\lambda}{2}\,,\,\hat{X}_\mu^L\right] + \frac{1}{g}\,\partial_\mu\,\epsilon^L\,\lambda \\
\delta \hat{X}_\mu^R &= i\left[\frac{\epsilon^R\,\lambda}{2}\,,\,\hat{X}_\mu^R\right] + \frac{1}{g}\,\partial_\mu\,\epsilon^R . \lambda \qquad (4.1) \\
\delta X_\mu^{L,o} &= \frac{1}{g_o}\,\partial_\mu\,\epsilon_o^L \\
\delta X_\mu^{R,o} &= \frac{1}{g_o}\,\partial_\mu\,\epsilon_o^R
\end{aligned}
$$

where,

$$
\epsilon^L\,\lambda = \sum_{k=1}^{8} \epsilon_k^L\,\lambda_k
$$

$$
\epsilon^R .\,\lambda = \sum_{k=1}^{8} \epsilon_k^R\,\lambda_k
$$

The currents are calculated using the Noethers theorem If the fields ψ_k transform as,

$$\psi_k \to \psi_k + \epsilon_i \, (T_i)_{kl} \, \psi_l \tag{4.2}$$

where $(T_i)_{kl}$ are matrices representing the group generators for the representation to which ψ belongs. The currents are given by,

$$J^i_\mu = - \frac{\partial L}{\partial(\partial_\mu \psi_k)} (T_i)_{kl} \, \psi_l \tag{4.3}$$

In the case when the transformed fields involve the derivatives of the group parameters, as in the case of gauge transformations, the currents can be easily calculated in the following way.[52] Let the fields ψ_k transform as,

$$\psi_k \to \psi_k + \epsilon_i \, \delta^{(i)} \, \psi_k + \partial_\mu \, \epsilon_j \, \chi^{(j)}_{\mu \, k} \tag{4.4}$$

Then the currents are given by,

$$J^i_\mu = - \frac{\delta L}{(\delta \, \partial_\mu \epsilon_i)} + \partial_\mu \, f^i{}_{\nu\mu} \tag{4.5}$$

where,

$$f^i_{\mu\nu} = \frac{\partial L}{\partial(\partial_\mu \psi_k)} \, \chi^{(i)}_{\nu \, k} \tag{4.6}$$

Using the Eq (4.5) the vector and axial vector currents $J_\mu^{V_k}$, $J_\mu^{A_k}$ are calculated from the Lagrangian. In terms of the 3 x 3 matrices

$$\begin{aligned} J^V_\mu &= \frac{1}{\sqrt{2}} \sum_{k=1}^{8} J_\mu^{V_k} \lambda_k \\ J^A_\mu &= \frac{1}{\sqrt{2}} \sum_{k=1}^{8} J_\mu^{A_k} \lambda_k \end{aligned} \tag{4.7}$$

the octet currents are given by,

$$J_\mu^V = \frac{1}{\sqrt{2}} (J_\mu^L - J_\mu^R)$$
$$J_\mu^A = \frac{1}{\sqrt{2}} (J_\mu^L - J_\mu^R) \qquad (4.8)$$

where,

$$J_\mu^{L,R} = -\frac{m_o^2}{g} \hat{X}_\mu^{L,R} - \frac{1}{g} \partial_\nu f_{\nu\mu}^{L,R} \qquad (4.9)$$

and $f_{\mu\nu}^{L,R}$ are given by,

$$f_{\mu\nu}^L = \hat{X}_{\mu\nu}^L + \frac{h_1}{2} (\hat{X}_{\mu\nu}^L MM^+ + MM^+ \hat{X}_{\mu\nu}^L) + h_2 M \hat{X}_{\mu\nu}^R M^+ + ih_o (D_\mu M D_\nu M^+ - D_\nu M D_\mu M^+)$$
$$f_{\mu\nu}^R = \hat{X}_{\mu\nu}^R + \frac{h_1}{2} (\hat{X}_{\mu\nu}^R M^+M + M^+M \hat{X}_{\mu\nu}^R) + h_2 M^+\hat{X}_{\mu\nu}^L M + ih_o (D_\mu M^+ D_\nu M - D_\nu M^+ D_\mu M) \qquad (4.10)$$

In the above expression the currents are written in terms of the original fields X_μ^L etc. and are now expressed in terms of the physical fields as defined in Chapter II.

4 2 Meson Decay Constants

The π_{l_2} and the K_{l_2} decay constants are defined by,

$$\langle 0 \mid J_\mu^{A_k}(0) \mid P^k(q) \rangle = \frac{iq_\mu}{(2\pi)^{3/2}\sqrt{2\omega_q}} F_{P_k} \qquad (4.11)$$

Using the expressions obtained for the currents we get,

$$F_\pi = \sqrt{2}\alpha Z_\pi^{-1/2} , \qquad F_K = (\alpha+\beta) Z_K^{-1/2} /\sqrt{2} \qquad (4.12)$$

The decay constant F_{S_K} of strange scalar meson S_K is defined similarly by,

$$< 0 \mid J^k_\mu (0) \mid S_K(q) > \; = \; - \frac{q_\mu}{(2\pi)^{3/2} \sqrt{2\omega_q}} \; F_{S_K} \qquad (4.13)$$

and we have,

$$F_{S_K} \; = \; (\alpha-\beta) \; Z_{S_K}^{-1/2}/\sqrt{2} \qquad (4.14)$$

The mesonic masses, the decay constants and the renormalization constants satisfy the Glashow - Weinberg relations[50]

$$Z_\pi^{1/2} \; F_\pi \; = \; Z_K^{1/2} \; F_K \; + \; Z_{S_K}^{1/2} \; F_{S_K}$$

$$Z_\pi^{-1/2} \; m_\pi^2 \; F_\pi \; = \; Z_K^{-1/2} \; m_K^2 \; F_K \; + \; Z_{S_K}^{-1/2} \; m_{S_K}^2 \; F_{S_K} \; .$$

For the value 0.84 for the parameter δ we predict

$$F_K/F_\pi \; = \; 1.04 \quad , \quad F_{S_K}/F_\pi \; = \; -0.14 \qquad (4.15)$$

We donot predict F_π since $\Gamma(\pi \to \mu\nu)$ was used to fix value of g as discussed in Chapter III

For the neutral vector meson decays into lepton pair the decay constants are defined by[51]

$$< 0 \mid J^{e\,m}_\mu \; (0) \mid V^k(q) > \; = \; \frac{\varepsilon^k_\mu}{(2\pi)^{3/2} \sqrt{2\omega_q}} \; \frac{m^2_{V_k}}{f_{V_k}} \qquad (4.16)$$

The f_ρ , f_ω , f_ϕ are given by,

$$f_\rho \; = \; Z_\rho^{-1/2} \; g \; , \quad f_\omega \; = 2g_Y/\sin \theta_Y \; , \quad f_\phi \; = 2g_Y/\cos \theta_Y$$

where,

$$g_Y = \frac{\sqrt{3}}{2} g/((\frac{m_o}{m_\phi} \cos\theta)^2 + (\frac{m_o}{m} \sin\theta)^2)^{1/2}$$

$$\tan\theta_Y = \frac{m_\phi}{m_\omega} \tan\theta \quad \text{and} \quad \tan 2\theta = 2\sqrt{2} .$$

The predicted values are,

$$f_\rho^2/4\pi = 3.1, \quad f_\omega^2/4\pi = 27.5 , \quad f_\phi^2/4\pi = 24.5$$

Experimentally we have,

$$f_\rho^2/4\pi = 1.9 \pm 0.19 , \quad f_\omega^2/4\pi = 14.0 \pm 2.8 ,$$

$$f_\phi^2/4\pi = 11\ 0 \pm 0.9 ,$$

obtained from the rates for leptonic decays[52] while the photo-production data gives[53]

$$f_\rho^2/4\pi = 2.5 - 5.2 , \quad f_\omega^2/4\pi = 30.0 \pm 7.4 ,$$

$$f_\phi^2/4\pi = 13.6$$

The predicted f_ρ and f_ω are close to the photo production result while f_ϕ agrees with neither of the two experimental results

Weinberg's first sum rule

$$F_\pi^2 = (\frac{m_\rho^2}{g_\rho})^2 (\frac{1}{m_\rho^2} - \frac{1}{m_{A_1}^2})$$

is satisfied in our model. The second sum rule is satisfied only if $h_2 = 0$ is assumed. Also we do not have KSRF

relation $m_\rho^2 = 2F_\pi^2 g_\rho^2$. Numerically we have,

$$m_\rho^2 / 2F_\pi^2 g_\rho^2 = 0.78$$

4.3 $\underline{K_{l_3}}$ <u>Form Factors</u>

The form factors for the K_{l_3} decay[55]

$$K^- \rightarrow \pi^0 + l^- + \nu$$

are defined by,

$$\langle \pi^0(p) \mid J_\mu^{K^{*-}}(0) \mid K^-(k) \rangle = - \frac{1}{(2\pi)^3} \frac{1}{\sqrt{4\omega_p\omega_p}}$$

$$\times \left[\frac{1}{2} (k+p)_\mu f_+ (q^2) + \frac{1}{2} (k-p)_\mu f_- (q^2) \right] \qquad (4.17$$

where $J_\mu^{K^{*-}}$ is the strangeness changing vector current and $q^2 = -(k-p)^2$.

For calculating the form factors we donot use the full current given by Equations (4.7) - (4.10) but use only first term - the term proportional to the X_μ field - which when expressed in terms of physical fields becomes,

$$J_\mu^{K^{*-}} = - \frac{m_o^2}{g} Z_{K^*}^{1/2} K_\mu^{*-} + iF_{S_K} \partial_\mu S_K^-$$

$$- \frac{i\, m_o^2 (Z_K Z_\pi)^{1/2}}{4g^2 \alpha(\alpha + \gamma)} (K^- \partial_\mu \pi^0 - \pi^0 \partial_\mu K^-) + \cdot \quad \cdot \qquad (4.18)$$

where we have omitted terms which do not contribute to K_{l_3} form factors The form factors $f_\pm (q^2)$ are given by,[42]

$$f_+(q^2) = \frac{2}{q^2+m_{K^*}^2}\left(\frac{m_o^2}{g} Z_{K^*}^{1/2}\right)\left(g^{(1)}_{KK\pi} + \frac{1}{2} g^{(2)}_{K^*K\pi} q^2\right)$$

$$+ \frac{m_o^2}{2g^2} \frac{(Z_K Z_\pi)^{1/2}}{\alpha(\alpha+\gamma)}$$

and

$$f_-(q^2) = \frac{-2}{q^2+m_{K^*}^2}\left(\frac{m_K^2 - m_\pi^2}{m_{K^*}^{2}}\right)\left(\frac{m_o^2}{g} Z_{K^*}^{1/2}\right)$$

$$\times\left(g^{(1)}_{K^*K\pi} - \frac{1}{2} g_2 m_{K^*}^2\right) - 2\left(\frac{g^{(1)}_{S_K K\pi} - m_{S_K}^2 g^{(2)}_{S_K K\pi}}{q^2 + m_{S_K}^2}\right) F_{S_K} \tag{4.19}$$

Expanding $f_\pm(q^2)$ in powers of q^2, $f_\pm(0)$ and $\lambda_\pm$ are defined by,

$$f_\pm(q^2) = f_\pm(0)\left(1 + \frac{\lambda_\pm q^2}{m_\pi^2}\right)$$

In our model $f_+(0)$ obeys the Glashow Weinberg relation

$$f_+(0) = \frac{F_K^2 + F_\pi^2 - F_{S_K}^2}{2F_K F_\pi} \tag{4.20}$$

$$\approx 0.99$$

The value of $f_+(0)$ is close to 1, the value in limit of exact SU(3) symmetry, in accordance with the Ademallo Gatto theorem[56] Other K_{l_3} decay parameters are

$$\xi \equiv f_-(0)/(f_+(0)) = -0.28$$
$$\lambda_+ = 0.021 \quad , \quad \lambda_- = -0.022 \tag{4.21}$$

Experimentally[55] the Dalitz plot analysis of K_{μ_3} decays gives $\xi = -0.15 \pm 0.5$. The μ-polarization data gives large and negative value $\xi = -1.45 \pm 0.70$ while $\Gamma_{\mu_3}/\Gamma_{e_3}$ branching ratio mesurements give $\xi = -0.53 \pm 0.18$ The overall fit to the three sets of experimental data gives $\xi = -0.85 \pm 0.20$, $\lambda_+ = 0.45 \pm 0.012$ The situation about λ_- is not clear and in most of the analyses it is assumed to be zero.

From the expression,

$$R = \frac{0.6457 + 3.8008\lambda_+ + 6.8120\lambda_+^2 + 0.1264\xi + 0.4757\xi\lambda_+ + 0.0192\,\xi^2}{1 + 3.6995\lambda_+ + 5.4777\lambda_+^2}$$

for the branching ratio $\Gamma(K \to \pi\mu\nu)/\Gamma(K \to \pi e\nu)$ we obtain

$$R = 0.640$$

which is to be compared with the experimental value 0.626 ± 0.19.

Using the soft pion method Callan and Treiman[57] and Mathur, Okubo and Pandit[58], have obtained the relation

$$f_+(q^2 = -m_K^2) + f_-(q^2 = -m_K^2) = F_K/F_\pi + \cdots$$

where the extra terms denoted by dots vanish in the limit of exact SU(2) X SU(2) symmetry. Numerically this relation is well satisfied in our model. The Ward identity techniques with PCAC and pole dominance assumptions give relation (4.20)[50]. This relation holds in our model as has been already remarked. Recently Dashen and Weinstein[59] derived the result,

$$\xi = -\frac{1}{2}\left(\frac{F_K}{F_\pi} - \frac{F_\pi}{F_K}\right) - \lambda_+\left(\frac{m_K^2 - m_\pi^2}{m_\pi^2}\right) \qquad (4.22)$$

An estimate of right hand side in our model gives $\xi = -0.284$ which is close to the value -0.28 (Eq. 4.21)

Many authors[60] have calculated the K_{l_3} form factors by using dispersion relations and soft pion techniques. The results for ξ and $\lambda_\pm$ depend on additional assumptions made about number of subtractions and on the values of M_{S_K} and F_K/F_π In the hard pion approach ξ , $\lambda_\pm$ can be calculated if further assumptions about the S_K meson mass and couplings are made[61]. The results in this case also depend on details of assumptions made.

Experimentally ξ is found to be negative and large As is obvious from Eq. (4.22) a large negative ξ may be obtained if F_K/F_π is large ($\simeq 1.3$). Small value of F_K/F_π may be cause of small negative value in the present work. Large F_K/F_π, however, implies large SU(3) breaking by the vacuum and this situation cannot be obtained in our model if the scalar mesons are to kept around 1000 MeV.

Brandt and Preparata[62] have obtained $\xi \simeq -1$ by using a modified version of PCAC hypothesis. However, Weinstein[63] has argued that their results depend not on modification of PCAC hypothesis but on large SU(3) violations.

It has been suggested by Schilcher[64] and by Arnowitt, Friedman and Nath[65] that a symmetry breaking term transforming as a component of (1,8) + (8,1) representation may be used to explain large negative ξ .

CHAPTER V

ISOSPIN BREAKING EFFECTS

So far we have studied the isospin conserving interactions only. In this chapter, we introduce non-electromagnetic isospin breaking by assuming $a_3 \neq 0$ in Eq.(2.6). The term $a_3 u_3$ breaks the isospin invariance explicitly and transforms as a component of $(3,3^*) + (3^*,3)$ representation[66]. The existence of an isospin violating piece in the Lagrangian has been postulated by many authors to explain the electromagnetic mass differences[67], the $\eta \to 3\pi$ decay[68] and the origin of Cabbibo angle[69]. Indeed if the only isospin violating effects are those coming from the interactions of photon e.m. mass differences of pseudo-scalar mesons should obey the relation[70]

$$m^2_{K^+} - m^2_{K^0} = m^2_{\pi^+} - m^2_{\pi^0}$$

which does not agree with the experiments. In the case of η-decays it was pointed out by Sutherland[71] that the amplitude vanishes in the soft pion limit if PCAC is assumed. To explain the η-decays it was proposed[68] that a term transforming as 3rd component of an isovector should be added to the Hamiltonian. The $a_3 u_3$ term has been studied by Gatto Sartori and Tonin[72] and by Cabbibo and Miani[73] in connection with the origin of Cabbibo angle.

The consistency conditions that VEV's of σ fields are zero are given by Eq (2.24) where now η is given by Eq. (5.1) and A is given by Eq (2.6) with $a_3 \neq 0$

The physical fields are defined and masses are obtained in the tree approximation as in Chapter II. The Eq. (2.24) now gives a relation between the parameters η_3 and a_3. Thus all the e.m. mass differences and coupling constants are functions of a single parameter which we take to be η_3

Nonvanishing of η_3 gives rise to the mass differences in various isospin multiplets. To the mass differences so calculated in the tree approximation the nontadpole contributions must be added These nontadpole contributions have been estimated by many authors.[67,74] The nontadpole contribution to $\Delta m_\pi (\equiv m_{\pi^+} - m_{\pi^0})$ is close to the experimental value while that for $\Delta m_K (\equiv m_{K^0} - m_{K^+})$ is not In the tree approximation in our model Δm_π turns out to be small, being of the order of η_3^2 while Δm_K is proportional to η_3. The total mass differences are given in Table VII for some values of η_3.

For $\delta' (\equiv \frac{(\alpha-\beta)}{2} \times 10^3) = - 4.0$, we have

$$m_{\rho^+} - m_{\rho^0} = 2.5 \text{ MeV} , \qquad m_{K^{*0}} - m_{K^{*+}} = 4.9 \text{ MeV}$$

No estimates about the nontadpole contributions to these mass differences are available.

Table VII Variation of the e.m. Mass Differences of Pseudoscalar Mesons and $\eta \to 3\pi$ Decay Parameters with δ'.

δ'	-3.6	-3.8	-4.0	-4.2	-4.4	Exp. value[a]
$(m_{\pi^+} - m_{\pi^0})^b$	4.97	5.03	5.08	5.14	5.21	4.604 ± 0.004
$(m_{K^0} - m_{K^+})^b$	3.31	3.62	3.94	4.25	4.57	3.94 ± 0 13
$\Gamma(\eta \to \pi^+\pi^-\pi^0)^c$	0.285	0.317	0.349	0.391	0.432	0.61 ± 0.16
$\Gamma(\eta \to 3\pi^0)^c$	0.252	0.280	0.310	0.342	0.375	0.90 ± 0 21
R	0.88	0.88	0.89	0.87	0.87	1.31 ± 0.21
α	0.10	0.10	0.11	0.12	0.12	-0.50 ± 0 04

a Experimental values are from Ref. 27, except for α which is from Ref 78

b In MeV. The values of nontadpole contributions have been taken from R H. Socolow, Ref. 67.

c In keV

In the presence of isospin violating interactions we can now estimate the rates for $\rho \to \eta\pi$ and $\omega \to \pi^+\pi^-$. For $\delta' = -4.0$ we have,

$$\Gamma(\rho \to \eta\pi) = 3.8 \text{ keV}$$

which is well within the experimental upper limit of a few MeV[27] The width for $\omega \to \pi^+\pi^-$, however, turns out to be as large as $\Gamma(\rho^0 \to \pi^+\pi^-)$ This is because when $\eta_3 \neq 0$ the $\rho - \omega$ mixing angle is large ($\theta = 45^0$) and independent of η_3. Therefore $\Gamma(\omega \to \pi^+\pi^-)$ cannot be reduced by choosing a lower value for η_3. This problem is due to the nonet structure of the terms proportional to h_1 and h_2 in the Lagrangian. These terms were essential to have $\omega - \phi$ mixing and a good fit to the spin one meson masses. One might attempt to cure this problem by writing terms having same structure as these terms but with different coupling constants for the octet and singlet fields but then the number of parameters becomes too large to fix them from spin one meson masses alone The correct value for $\Gamma(\omega \to \pi^+\pi^-)$ will be obtained in a different model in Chapter VI where vector mesons acquire masses through Higgs-Kibble mechanism only and the $\omega - \phi$ mixing is induced by photon.

The values of parameters characterizing the relative strengths of isospin breaking and SU(3) breaking in the Lagrangian and by the vacuum are,

$$d \equiv a_3/a_8 = (0.034 - 0.043)$$

$$\zeta \equiv \eta_3/\eta_8 = (0.039 - 0.050)$$

The isospin breaking effects have been extensively studied in context of electromagnetic mass differences[67] and determination of Cabibbo angle[72,73] Cicogna et al [75] obtained $d = (0.017 - 0.026)$ $\zeta = (0.023 - 0.030)$ while Parisi and Testa[76] obtained $d = 0.027$ and $\zeta = 0.032$ using Ward identities to study e.m. mass differences of pseudoscalar mesons. The value $d = 0.05$ obtained from Cabibbo angle studies by Oakes[77] is in the range obtained in the present work. Other values[78] $d = 0.017$ and $d = 0.88/137$, however, are not in the range obtained in our model.

5.2 $\eta \rightarrow 3\pi$ Decays

For the $\eta \rightarrow 3\pi$ decays the amplitude, m, is defined by,

$$\langle 3\pi, k_1, k_2, k_3 | S | (q) \rangle$$

$$= (2\pi)^4 \, \delta(q - \sum_{i=1}^{3} k_i) \frac{m}{(2\pi)^6 \sqrt{16\omega_1\omega_2\omega_3\omega_q}}$$

and the rate is obtained by integrating square of the amplitude as discussed in Chapter III.

The slope, α, of the Dalitz plot for the $\eta \rightarrow \pi^+\pi^-\pi^0$ process is defined by,

$$m_{+-0}(y) = m(0)\,(1 + \alpha y + O(y^2))$$

where m_{+-0} is the amplitude for $\eta \rightarrow \pi^+\pi^-\pi^0$ and

$$y = \frac{3T_{\pi^0}}{Q}, \quad Q = m_\eta - 2m_{\pi^+} - m_{\pi^0}.$$

and T_{π^0} is the kinetic energy of the π^0

The tree diagrams which contribute to the η decays are shown in Figure 3. Using the effective vertices the rates for $\eta \to \pi^+\pi^-\pi^0$ and $3\pi^0$ modes, the branching ratio and the slope parameter α are calculated and are given in Table VII along with the e.m mass differences for some values of δ'. In the range of values of δ' determined from the pseudoscalar mass differences the rates and the slope parameters are lower than the observed values.

Recently $\eta \to 3\pi$ decays have been discussed extensively. In connection with the $(3,3^*) + (3^*,3)$ symmetry breaking scheme it has been claimed that the same set of parameters can ~~not~~ explain both pseudoscalar e m mass differences and decays.[80] Dittner, Dondi and Eliezer[81] have recently shown that η-decays cannot be adequately described in any of the simple symmetry breaking schemes like $(3,3^*) + (3^*,3)$, $(6, 6^*) + (6^*,6)$ or $(8, 8)$ for the total interaction Hamiltonian. Spivack and Rosen[82], from their study of η-decay, have concluded that the mass term for the pseudoscalar mesons must contain the $(3,3^*) + (3^*,3)$ and at least one other representation and that the kinetic Lagrangian must break chiral SU(3)

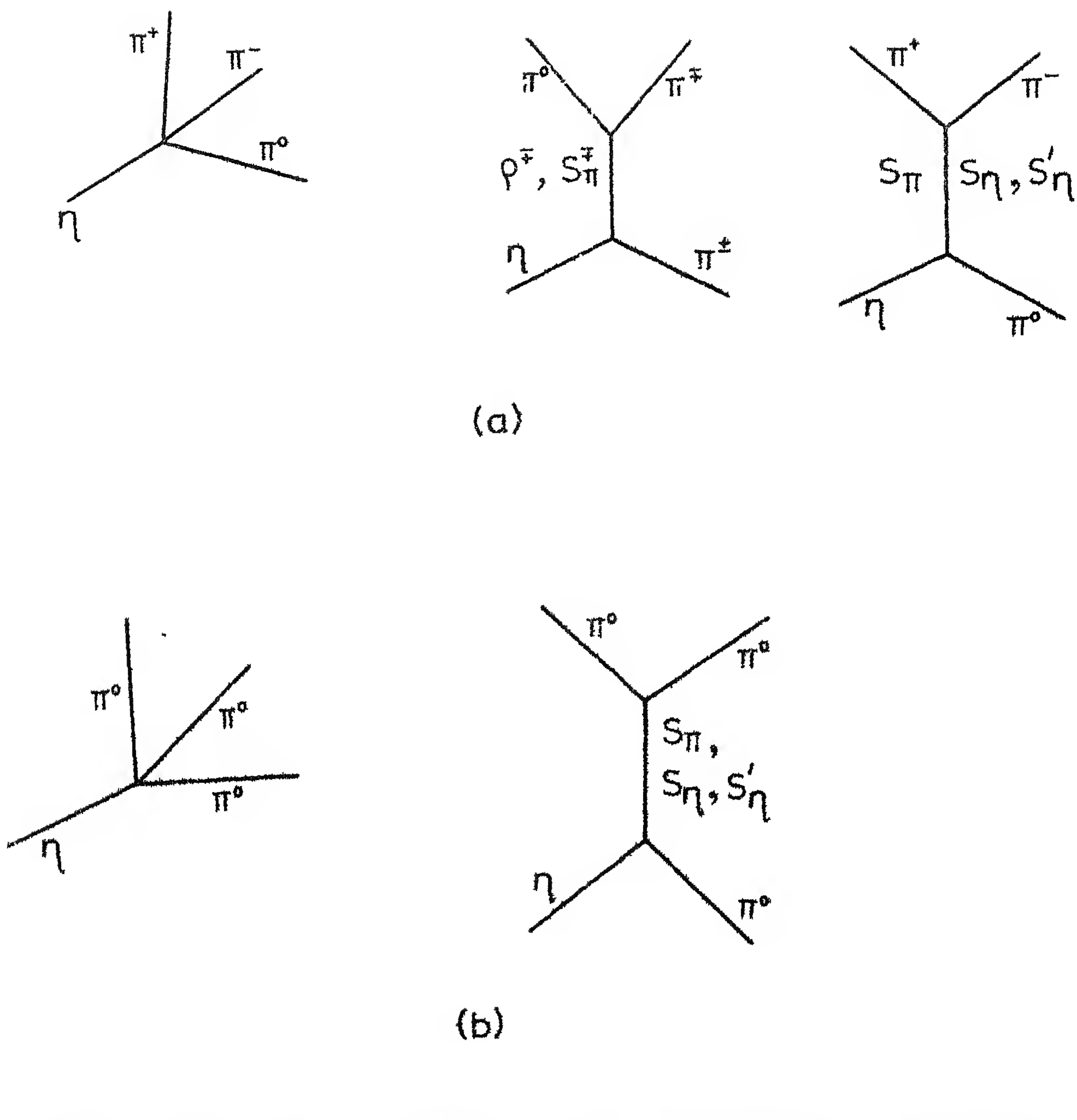


Fig. 3 Feynman graphs for η decays (a) $\eta \longrightarrow \pi^+ \pi^- \pi^0$
(b) $\eta \longrightarrow 3\pi^0$.

CHAPTER VI

A RENORMALIZABLE MODEL WITH SPONTANEOUS SYMMETRY BREAKING

So far we have been studying an effective Lagrangian model of chiral symmetry In this model studied in Chapters II to V, the Lagrangian was not fully gauge invariant and also non-renormalizable. In this chapter we present a model which is fully gauge invariant to start with and spontaneous symmetry breaking is used to obtain all symmetry breaking effects including physical masses for the gauge fields.[24] This is achieved by introducing an appropriate set of scalar fields called Higgs-Kibble scalars.[23] We show that the Glashow - Weinberg[15] and Gell-Mann, Oakes, Renner[15] type of symmetry breaking for hadrons is obtained in the limit in which the masses of some of the Higgs-Kibble scalar mesons become infinite.

6.1 The Lagrangian and Particle Masses

The Lagrangian is again written only for the spin $0^{\pm}$ mesons and the vector and axial vector gauge fields. From a study of mesonic masses it is concluded that a ninth axial vector meson cannot be included in the model and hence we donot insist on invariance under ninth axial transformations. Thus Lagrangian is invariant under $SU(3) \times SU(3) \times U(1)_{V_0}$. For the Higgs-Kibble scalars we are forced to extend the

definition of charge and hypercharge by introducing additional quantum numbers K_Q and K_Y The U(1) symmetry generated by K_Q is also gauged and as a consequence of which we get couplings of photon in a natural way. The effect of these couplings on $\rho - \omega$ and $\rho - \phi$ mixings is found to be in agreement with the experiments The lepton pair decay constants of ρ^0, ω^0 and ϕ^0 are also calculated

The Higgs-Kibble scalar fields are chosen to be triplets $\phi_L^{(a)}$, $\phi_R^{(a)}$ (a = 1,2,3) and are assumed to transform as (3,1) and (1,3) respectively Since these fields are assumed to have non zero VEV's and none of these has zero charge and hypercharge we assume modified expressions for charge and hypercharge

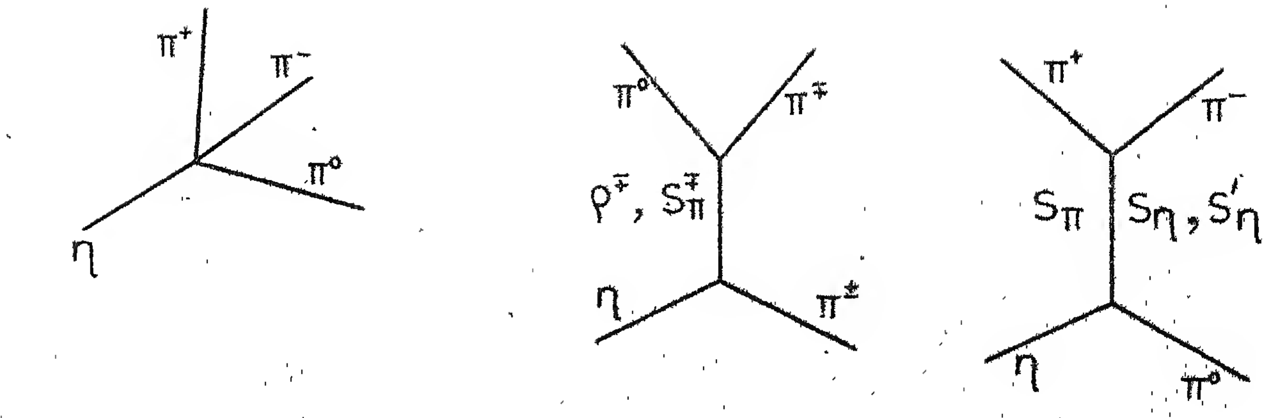
$$Q = F_3 + \frac{1}{\sqrt{3}} F_8 + K_Q$$

$$Y = \frac{2}{\sqrt{3}} F_8 + K_Y$$

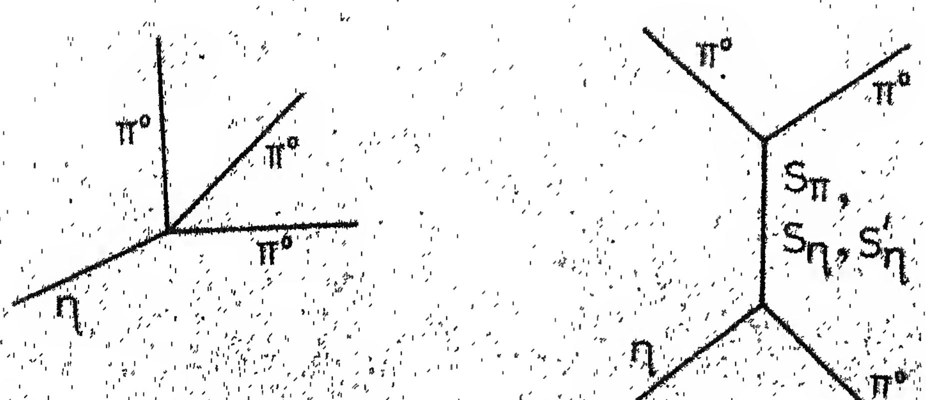
where the two new quantum numbers K_Q and K_Y are assumed to be zero for observed hadrons and have the following values for the scalar fields:

$$\phi_{L,R}^{(1)} \quad K_Y = -1/3 \;, \quad K_Q = -2/3$$
$$\phi_{L,R}^{(2)} \quad K_Y = -1/3 \;, \quad K_Q = 1/3$$
$$\phi_{L,R}^{(3)} : \quad K_Y = 2/3 \;, \quad K_Q = 1/3$$

The Lagrangian for the gauge fields and the $\phi_{L,R}$ fields is



(a)



(b)

Fig. 3 Feynman graphs for η decays (a) $\eta \longrightarrow \pi^+ \pi^- \pi^0$
(b) $\eta \longrightarrow 3\pi^0$.

and define new fields,

$$M' = M - \langle M \rangle_0$$

which have zero VEVs. The Lagrangian is written in terms of the M' fields and various mixings are removed as discussed in the appendix and physical fields are defined as in Chapter II

The masses of vector and axial vector mesons are given by (assuming $f_1 = f_2$),

$$m^2_{\rho^\pm} = g^2 f_1^2 ,$$

$$m^2_{K^*} = \frac{1}{2} g^2 (f_1^2 + f_3^2) + \frac{1}{2} g^2 (\alpha - \gamma)^2$$

$$m^2_{A_1} = g^2 f_1^2 + 2g^2 \alpha^2 ,$$

$$m^2_{K_A} = \frac{1}{2} g^2 (f_1^2 + f_3^2) + \frac{1}{2} g^2 (\alpha + \gamma)^2 ,$$

$$m^2_{A_8} = \frac{1}{3} g^2 (f_1^2 + 2f_3^2) + \frac{2}{3} g^2 (\alpha^2 + 2\gamma^2)$$

The mass term for the neutral vector mesons is,

$$- \frac{1}{2} f_1^2 \left(\frac{g}{\sqrt{2}} V_\mu^3 + \frac{g}{\sqrt{6}} V_\mu^8 + \frac{g_0}{\sqrt{3}} V_\mu^0 - \frac{2\sqrt{2}}{3} g' V_\mu' \right)^2$$

$$- \frac{1}{2} f_1^2 \left(- \frac{g}{\sqrt{2}} V_\mu^3 + \frac{g}{\sqrt{6}} V_\mu^8 + \frac{g_0}{\sqrt{3}} V_\mu^0 + \frac{\sqrt{2}}{3} g' V_\mu' \right)^2$$

$$- \frac{1}{2} f_3^2 \left(- \frac{2g}{\sqrt{6}} V_\mu^8 + \frac{g_0}{\sqrt{3}} V_\mu^0 + \frac{\sqrt{2}}{3} g' V_\mu' \right)^2$$

One of the eigenvalues of the mass matrix is zero. The corresponding field,

$$A_\mu = \frac{1}{N} \left(- \frac{x}{\sqrt{2}} V_\mu^3 - \frac{x}{\sqrt{6}} V_\mu^8 + V_\mu' \right)$$

is identified with photon. Here we have,

$$x = \sqrt{2}\, g'/g$$

and

$$N^2 = (2x^2 + 3)/3$$

The other three nonzero eigenvalues are squares of masses of ρ, ω and ϕ.

The coupling constant of hadronic e m. current $(J_\mu^3 + \frac{1}{2} J_\mu^Y)$ is identified with e to get,

$$\frac{e^2}{4\pi} = \frac{g^2}{4\pi} \left(\frac{x^2}{2N^2} \right) = \frac{1}{4\pi} \left(\frac{g'^2}{N^2} \right)$$

The masses of scalar and pseudoscalar bosons same as in Chapter II with $\frac{1}{2} \epsilon_1 f_1^2$ and $\frac{1}{2} \epsilon_3 f_3^2$ now being identified with the parameters a and c respectively. The expressions for renormalization Constants for the spin zero mesons, however, are different and are given by,

$$Z_\pi = 1 + 2\alpha^2/ f_1^2$$

$$Z_K = 1 + (\alpha + \gamma)^2/(f_1^2 + f_3^2)$$

$$Z_{S_K} = 1 + (\alpha - \gamma)^2/(f_1^2 + f_3^2)$$

The m_η^2 and $m_{\eta'}^2$ are now eigenvalues of the matrix $K^{-1/2} M_P^2 K^{1/2}$ where,

$$(M_P^2)_{88} = \mu_0^2 \left(1 + \frac{\nu_1}{3} (\alpha^2 + 2\gamma^2)\right) - \frac{\lambda}{3} (4\alpha - \gamma)$$

$$(M_P^2)_{00} = \mu_0^2 \left(1 + \frac{\nu_1}{3} (2\alpha^2 + \gamma^2)\right) - \frac{2\lambda}{3} (2\alpha + \gamma)$$

$$(M_p^2)_{08} = \frac{2}{3}(\alpha - \gamma)(\mu_0^2 \nu_1 (2\alpha^2 + \gamma^2) - \lambda)$$

$$(K)_{88} = 1 - 2g^2 (\alpha + 2\gamma)^2 / 3m_{A_8}^2$$

$$(K)_{00} = 1 - 2g^2 (\alpha - \gamma)^2 / 3m_{A_8}^2$$

$$(K)_{08} = -2g^2 (\alpha - \gamma)(\alpha + 2\gamma) / 9m_{A_8}^2$$

The parameter $\delta(= \alpha/\gamma)$ was varied in a suitable range. For the value $\delta = 0\ 81$ the scalar meson masses were found to be close to experimental values The parameters gf_1, gf_3, $g\alpha$ were fixed by taking m_ρ , m_{K^*} and m_{A_1} (= 1070 MeV) as input. g_0/g, and g'/g were fixed by using $(m_\omega{}^2 + m_\phi{}^2)$ and $\frac{e^2}{4\pi}$ as input The predicted values for the rest of the spin are meson masses are,

$$m_\omega = 784\ 5 \text{ MeV} \quad (783.9 \pm 0.3)$$

$$m_\phi = 1017.4 \text{ MeV} \quad (1019.5 \pm 0.3)$$

$$m_{K_A} = 1239 \text{ MeV} \quad (1242 \pm 10)$$

$$m_{A_8} = 1290 \text{ MeV} \quad (1286 \pm 4)$$

which compare well with the experimental values given in the brackets above

The pseudoscalar meson masses depend only on the parameters $\nu_0 = 1 - 4\nu_2 (\alpha + 2\gamma^2)$, ν_2 and λ which were fixed by taking m_π^2 , m_K^2 and $(m_\eta^2 + m_{X^0}^2)$ as input. The calculated values of m_η and m_{X^0} are,

$$m_\eta = 529\ (548.8 \pm 0.8), \quad m_{X^0} = 969\ (957.5 \pm 0.8)$$

The m_{S_π} and m_{S_K} are predicted to be 1018 MeV and 1070 MeV respectively and are close to the masses of the $\pi_N(1016)$ and K_N (1080 - 1200) resonances The masses of the eighth and singlet scalar mesons depend on an additional parameter ν_2 which cannot be determined accurately. When ν_2 is varied from 0.0 to 1.0 the masses m_{S_η} and $m_{S'_\eta}$ vary from 400 MeV to 934 MeV and from 1145 MeV to 1434 MeV The resonances which may be identified with these scalar mesons are one of $\sigma(410)$ and $\epsilon(700)$ and the S^* (1060) respectively

If the invariance under 9th axial transfer motions is required the parameter λ must be set equal to zero. The ninth axial vector field can then be introduced as a gauge field In this case, however, it is not possible to obtain reasonable values of masses for η and X^0. The masses of η , X^0, A_8 and A_9 now depend on the coupling constant of the ninth axial vector gauge field also. If m_{A_8} and m_{A_9} are to be kept around the values for two of the three possible experimental candidates M (950), D(1280), E(1422), m_η remains very low ($\sim$ 200 MeV) To explain m_η the term (det M + det M^+) seems to be necessary.

6.2 $\rho - \omega$, and $\rho - \phi$ Mixings.

We first note that in this model the photon has been naturally included which was not possible in the model I. The most important consequence of this is the $\rho - \omega$ and $\rho - \phi$ mixing induced by V'_μ. These mixings are absent when $g' = 0$.

Thus the field V_μ^3 is no longer a pure ρ^0 field but is a linear combination of ρ^0, ω, ϕ and photon fields. Diagonalizing the mass matrix the physical fields are obtained as linear combinations of V_μ^3 , V_μ^8 , V_μ^0 ,and V_μ' and these in turn can therefore, be expressed as linear combinations of ρ , ω , ϕ and photon fields As a test of $\rho - \omega$ and $f - \phi$ mixings thus obtained we estimate $\Gamma(\omega \to 2\pi)$ and $\Gamma(\phi \to 2\pi)$ widths to get,

$$\Gamma(\omega \to 2\pi) = 0.087 \text{ MeV}$$
$$\Gamma(\phi \to 2\pi) = 0.73 \text{ keV}$$

where we have used $\Gamma(\rho \to 2\pi) = 125$ MeV to fix value of $g^2/4\pi$. The predicted value of $\Gamma(\omega \to 2\pi)$ agrees with the experimental value (0.10 ± 0.03) MeV. The experimental situation about $\Gamma(\phi \to 2\pi)$ is not clear The predicted value is well within the experimental upper limit of 2.4 keV obtained from $e^+ - e^-$ colliding beam experiments

The field V_μ' will also couple to the e.m. currents of fields which are singlets under hadronic SU(3) X SU(3). Thus with leptons the coupling will be of the form $g' V_\mu'(x) l_\mu^{e.m.}(x)$ Now V_μ' is expressed as linear combination of the neutral vector meson fields ρ, ω , ϕ and A_μ . The matrix elements for the vector meson decay into lepton pairs are directly obtained. Defining f_{V_i}, the neutral vector meson decay constants, as in Eqn. (4.16) we obtain,

$$f_A^2/4\pi = 12\ 6\ , \qquad f_\phi^2/4\pi = 12.2$$

for $f_\omega^2/4\pi = g^2/4\pi = 2\ 13$ These values are in better agreement with experimental values obtained from leptonic decay rates[51] and photoproduction data[52] except that $f_\omega^2/4\pi$ from photoproduction is 30 ± 7.4 which is larger than the above value.

6.3 Conclusion

In the following we make some concluding remarks

(i) In the second model we have not calculated the vector and axial vector meson decays, meson-meson scattering lengths and effective ranges, K_{l_3} decay form factors, and widths for $\eta \to 3\pi$. The effective Lagrangian for hadrons is same as the basic Lagrangian in the case of first model except for the non-minimal coupling terms. Therefore the axial vector meson decay widths are not expected to be in agreement with experiments. As the model is renormalizable the higher order corrections to these may be estimated and compared with experiments. The result, K_{l_3} form factors meson meson scattering parameters and widths for $\eta \to 3\pi$ decays are expected to be same in two models

(ii) In the second model the auxiliary scalar fields, which do not get eliminated by Higgs-Kibble mechanism, do not play any role. Their masses were assumed to be infinitely large.

It would be interesting if they can be identified with some of the physical fields and their interactions with hadrons are studied.

(iii) As all the symmetry breaking effects including gauge field masses have been obtained through the Higgs-Kibble mechanism and the basic Lagrangian is gauge invariant the second model is renormalizable[25] and tree unitary.[83]

(iv) As the second model is renormalizable higher order calculations may be done and corrections to various processes may be calculated Higher order calculations have been done in spontaneously broken gauge theories of weak and e.m. interactions [84] In the present case of strong interactions just perturbation scheme is not meaningful In the past perturbative calculations supplemented by summation techniques, like Pade approximants, have been used to get physically interesting results [85] A similar approach could be used in the case of second model.

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APPENDIX A

FIELD MIXINGS AND RENORMALIZATIONS

In this appendix we consider the mixing problem in various multiplets. First we consider the mixing of spin-zero and spin-one mesons. When the Lagrangian is written in terms of new fields χ , ϕ, .. . There is a mixing of type $Y^V_\mu \cdot \partial_\mu \chi$ and $Y^A_\mu \cdot \partial_\mu \phi$. For the sake of definiteness we consider first the mixing between axial vectors and pseudoscalars and the vector-scalar mixing can be treated in a similar manner.

Let the term giving the A-P mixing be written as

$$Y^{A\,i}_\mu \quad \xi_{ij} \quad \partial_\mu \quad \phi_j \tag{A.1}$$

To remove this mixing we substitute

$$Y^{A\,i}_\mu = y^{A\,i}_\mu - C^A_{ij} \quad \partial_\mu \phi_j$$

The term (A.1) together with the mass term for Y^A_μ —— $(-\frac{1}{2})(M^2_A)_{ij}$ $Y^{A\,i}_\mu Y^{A\,j}_\mu$ —— becomes

$$- \frac{1}{2} (M^2_A)_{ij} (y^{A\,i}_\mu - C^A_{ik} \partial_\mu \phi_k) (y^{A\,j}_\mu - C^A_{jl} \partial_\mu \phi_l)$$

$$+ (y^{A\,i}_\mu - C^A_{ik} \partial_\mu \phi_k) \quad \xi_{ij} \partial_\mu \phi_j$$

$$= - \frac{1}{2} (M^2_A)_{ij} y^{A\,i}_\mu y^{A\,j}_\mu + [(M^2_A)_{ij} C^A_{jk} + \xi_{ik}] y^{A\,i}_\mu \partial_\mu \phi_k$$

$$+ \partial_\mu \phi_k \partial_\mu \phi_j C^A_{jk} \xi_{ij} - \frac{1}{2} (M^2_A)_{ij} (C^A)_{ik} (C^A)_{jl} \partial_\mu \phi_k \partial_\mu \phi_l \tag{A.2}$$

The terms of type $y^A_\mu \cdot \partial_\mu \phi$ will be absent if

$$C^A_{ij} = -(M_A^{-2})_{ik}\, \xi_{kj}$$

Thus there is an extra kinetic energy term for ϕ fields in Eq.(A.2) which, on using above equation, becomes,

$$+\frac{1}{2}\, \partial_\mu \phi_k \, (\xi^T M_A^{-2}\, \xi)_{kj} \quad \partial_\mu\, \phi_j$$

This equation has been used in writing Eq (2.28).

After the mixing between spin one and spin zero fields has been removed there is mixing between 8-th and 0-th (8-th 0-th and 3rd) components in each multiplet when $n_3 = 0 (n_3 \neq 0)$. To illustrate how this mixing is removed and the physical fields are defined we consider the case of pseudoscalar mesons Other cases can be treated similarly. Let the kinetic energy and the mass term for these fields be written as

$$-\frac{1}{2} \quad \partial_\mu\, \phi^T\, K\, \partial_\mu \phi - \frac{1}{2}\, \phi^T M^2\, \phi \tag{A.3}$$

where ϕ is a column with ϕ^8 and ϕ^0 (ϕ^8, ϕ^0 and ϕ^3) as components and K and M^2 are the kinetic energy and mass squared matrices.

We first diagonalize the matrix K by an orthogonal matrix U_K

$U_K\, K\, U_K^T$ = a diagonal matrix, say K_D,

and define new fields by,

$$\phi' = U_K\, \phi \tag{A.4}$$

In terms of the fields

$$\emptyset'' = K_D^{1/2}\, \emptyset' \tag{A.5}$$

The expression (A.3) becomes,

$$-\frac{1}{2}\partial_\mu \emptyset''^{\,T} \partial_\mu \emptyset'' - \frac{1}{2}\emptyset''^{\,T} M_r^2\, \emptyset'' \tag{A.6}$$

where M_r^2 is renormalized mass squared matrix given by

$$M_r^2 = K_D^{1/2}\, U_K\, M^2\, U_K^T\, K_D^{-1/2}$$

M_r^2 is diagonalized by an orthogonal transformation U_M to get squares of renormalized masses of the physical pseudoscalar fields defined by,

$$P = U_M\, \emptyset'' \tag{A.7}$$

The original $\emptyset$ fields are now easily expressed in terms of physical fields, P, by using Eqs. (A.4), (A.5) and (A.7) to get

$$\emptyset = U_K^T\, K_D^{-1/2}\, U_M^T\, P$$

APPENDIX B

EFFECTIVE THREE AND FOUR POINT VERTICES

In this appendix we give the structure of some strong interaction vertices for the first model

We define C_π and C_K through the eqns ,

$$C_\pi = 1/\sqrt{2}\, g\, \alpha\, Z_\pi^{1/2}$$

$$C_K = \sqrt{2}/g(\alpha + \gamma)\, Z_K^{1/2}$$

Throughout this appendix θ will denote the octet singlet mixing angle for the scalar mesons For vector and axial vector mesons we define

$$V^k_{\mu\nu} \equiv \partial_\mu V^k_\nu - \partial_\nu V^k_\mu$$

and

$$A^k_{\mu\nu} \equiv \partial_\mu A^k_\nu - \partial_\nu A^k_\mu$$

Particle symbols will be used to denote corresponding fields

(i) VPP Vertex

$$\begin{aligned} L\,(VPP) = & -g^{(1)}_{\rho\pi\pi}\, \vec{\rho}_\mu \cdot \vec{\pi} \times \partial_\mu \vec{\pi} - g^{(2)}_{\rho\pi\pi}\, \vec{\rho}_{\mu\nu} \cdot (\partial_\mu \vec{\pi} \times \partial_\nu \vec{\pi}) \\ & + [i\sqrt{2}\, g^{(1)}_{K^*K\pi}\, K^{*+}_\mu\, \hat{\pi}\, \partial_\mu K + \\ & + i\sqrt{2}\, g^{(2)}_{K^*K\pi}\, K^{*+}_{\mu\nu}\, \partial_\mu \hat{\pi}\, \partial_\nu K + h.c.] \\ & + i g^{(1)}_{\phi K\bar{K}}\, \phi_\mu (K^+ \overleftrightarrow{\partial}_\mu K) + i g^{(2)}_{\phi K\bar{K}}\, \phi_{\mu\nu}\, \partial_\mu K^+\, \partial_\nu K + \ldots \end{aligned}$$

where $\hat{\pi} = \vec{\tau} \cdot \vec{\pi}/\sqrt{2}$

$$g^{(1)}_{\rho\pi\pi} = g\ Z_\pi\ Z_\rho^{1/2}\ (m_o^2/4g^2\alpha^2)$$

$$g^{(2)}_{\rho\pi\pi} = (g/2)\ C_\pi^2\ Z_\rho^{-1/2}\ (1 + 4h_o\ g\ \alpha^2\ Z_\rho)$$

$$g^{(1)}_{K^*K\pi} = g(Z_\pi\ Z_K\ Z_{K^*})^{1/2}\ (m_o^2/4g^2\ \alpha(\alpha+\gamma))$$

$$g^{(2)}_{K^*K\pi} = (g/4)\ C_K\ C_\pi\ Z_{K^*}^{-1/2}\ (1+2h_o\ g\alpha(\alpha+\gamma)\ Z_{K^*})$$

$$g^{(1)}_{\phi K\bar{K}} = -g\ Z_K\ Z_\phi^{1/2}\ (m_o^2/g^2(\alpha+\gamma)^2)$$

$$g^{(2)}_{\phi K\bar{K}} = -g\ C_K^2\ Z_\phi^{-1/2}\ (1 + h_o\ (\alpha+\gamma)^2\ Z_\phi)$$

(ii) <u>AVP Vertex</u>

$$L(AVP) = i\ g^{(1)}_{A_1\rho\pi}\vec{A}_{1\mu\nu}\ \partial_\mu\vec{\pi}\times\vec{\rho}_\nu + ig^{(2)}_{A_1\rho\pi}\ \vec{\rho}_{\mu\nu}.\vec{A}_{1\mu}\times\partial_\nu\vec{\pi}$$
$$+ \Big[\sqrt{2}i\ g^{(1)}_{K_AK^*\pi}\ K^{*+}_\nu\partial_\mu\ \hat{\pi}\ K_{A\mu\nu}$$
$$+ \sqrt{2}i\ g^{(2)}_{K_AK^*\pi}\ K^+_{A\mu}\partial_\nu\hat{\pi}\ K^*_{\mu\nu} + h.c\ \Big] + ..$$

$$g^{(1)}_{A_1\rho\pi} = -g\ C_\pi\ (Z_\rho/Z_{A_1})^{1/2}$$

$$g^{(2)}_{A_1\rho\pi} = -g\ C_\pi\ (Z_{A_1}/Z_\rho)^{1/2}\ (1 + 4h_o\ g\ \alpha^2\ Z_\rho)$$

$$g^{(1)}_{K_AK^*\pi} = (g/2)\ C_\pi\ (Z_{K^*}/Z_{K_A})^{1/2}\ (1+2h_o g\alpha\ (\alpha-\gamma)\ Z_{K^*})$$

$$g^{(2)}_{K_AK^*\pi} = -(g/2)\ C_\pi\ (Z_{K_A}/Z_{K^*})^{1/2}\ (1+2\ h_o\ g\ \alpha\ (\alpha+\gamma) Z_{K_A})$$

(iii) <u>SPP Vertex</u>

$$L(SPP) = \sum_{i=0,8} \Big[g^{(1)}_{\sigma_i \pi\pi}\ \sigma_i\ \partial_\mu \vec{\pi}\ \partial_\mu \vec{\pi} + g^{(2)}_{\sigma_i \pi\pi}\ \sigma_i\ \vec{\pi}\ \vec{\pi}$$

$$+ g^{(1)}_{\sigma_i K\overline{K}}\ \sigma_i\ \partial_\mu K^+\ \partial_\mu K + g^{(2)}_{\sigma_i K\bar{K}}\ \sigma_i\ K^+ K \Big] + \cdots$$

$$\sqrt{2}\ g^{(1)}_{\sigma_8 \pi\pi} = g^{(1)}_{\sigma_0 \pi\pi} = -\ 2g^2\ \alpha\ C_\pi/\sqrt{3}$$

$$\sqrt{2}\ g^{(2)}_{\sigma_8 \pi\pi} = g^{(2)}_{\sigma_0 \pi\pi} = -\ M_\pi^2/2\sqrt{3}\ \alpha^2$$

$$2\sqrt{2} g^{(1)}_{\sigma_8 K\overline{K}} = g^{(1)}_{\sigma_0 K\overline{K}} = 2g^2(\alpha+\gamma)\ C_K^2/\sqrt{3}$$

$$2\sqrt{2} g^{(2)}_{\sigma_8 K\overline{K}} = g^{(2)}_{\sigma_0 K\overline{K}} = 2\ M_K^2/\sqrt{3}\ (\alpha + \gamma)$$

$$+ g_{KK}^{(1)} \left[(\partial_\mu K^+ \partial_\nu K)^2 - (\partial_\mu K^+ \partial_\mu K)^2\right]$$

$$+ g_{KK}^{(2)} (\partial_\mu K^+ \partial_\nu K - \partial_\nu K^+ \partial_\mu K)^2$$

$$+ g_{KK}^{(3)} \left[(K^+ \partial_\mu K)^2 - (\partial_\mu K^+ K)^2 - (K^+ K)(\partial_\mu K^+ \partial_\mu K)\right]$$

$$+ g_{KK}^{(4)} (K^+ \partial_\mu K)(\partial_\mu K^+ K) + g_{KK}^{(5)} (K^+ K)^2 + .$$

$$g_{\pi\pi}^{(1)} = -g^2\, C_\pi^4\, Z_\rho^{-1}\, (1+8h_o g\, \alpha^2 Z_\rho)/4$$

$$g_{\pi\pi}^{(2)} = g^2\, Z_\pi (Z_\pi - 4)(m_o^2/96 g^2 \alpha^4)$$

$$g_{\pi\pi}^{(3)} = M_\pi^2\, Z_\pi / 48\, \alpha^2$$

$$g_{K\pi}^{(1)} = g^2\, C_K^2\, C_\pi^2\, Z_{K^*}^{-1}\, (1+4\, h_o\, g\alpha\, (\alpha+\gamma))/4$$

$$g_{K\pi}^{(2)} = g^2\, C_K^2\, C_\pi^2\, Z_\rho^{-1}\, (1+2\, h_o\, g\, (4\alpha^2 + (\alpha+\gamma)^2))/2$$

$$g_{K\pi}^{(3)} = 3(2Z_\pi + Z_K - Z_K Z_\pi)\, g_{K\pi}$$

$$g_{K\pi}^{(4)} = -2(Z_\pi + Z_K - Z_K Z_\pi)\, g_{K\pi}$$

$$g_{K\pi}^{(5)} = (Z_K Z_\pi - 4Z_\pi)\, g_{K\pi}$$

$$g_{K\pi}^{(6)} = (Z_K Z_\pi - 4Z_K)\, g_{K\pi}$$

$$g_{K\pi}^{(7)} = (3a+c)\,(3\alpha+\gamma)\, Z_K Z_\pi / 96\alpha^2 (\alpha+\gamma)^2$$

where,

$$g_{K\pi} = m_o^2/96\, g^2 \alpha^2 (\alpha+\gamma)^2$$

$$g_{KK}^{(1)} = g^2\, C_K^4\, Z_\rho^{-1}\, (1+2\, h_o\, g(\alpha+\gamma)^2\, Z_\rho)/2$$

$$g_{KK}^{(2)} = g^2 C_K^4 Z^{-1} (1+2 h_o g(\alpha+\gamma)^2 Z_\phi)/8$$

$$g_{KK}^{(3)} = g_{KK}^{(4)} = (4-Z_K) m_o^2 Z_K^3/6g^2(\alpha+\gamma)^4$$

$$g_{KK}^{(5)} = M_K^2 Z_K/6(\alpha+\gamma)^2$$